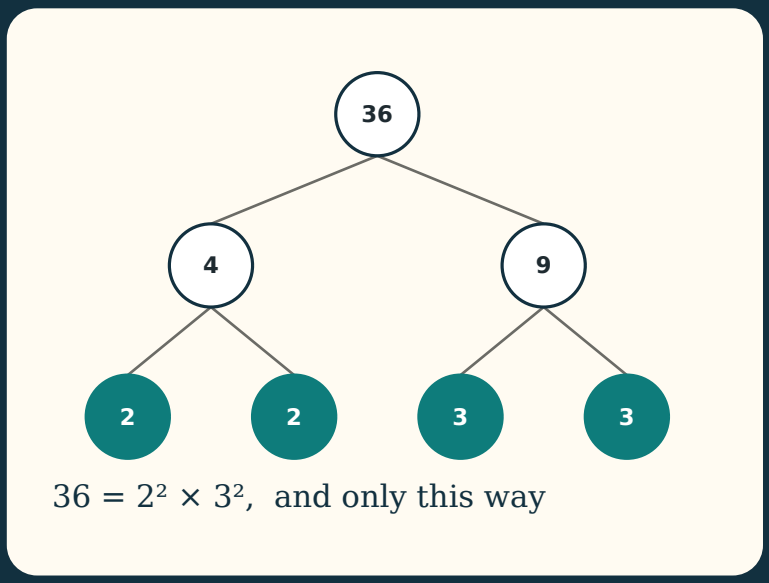
MATHS WITHOUT GUESSWORK

Real Numbers

Understand the Concept.
Recognise the Method.
Solve Independently.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27



$36 = 2^2 \times 3^2$, and only this way

Core topics: the Fundamental Theorem of Arithmetic · Proofs of irrationality

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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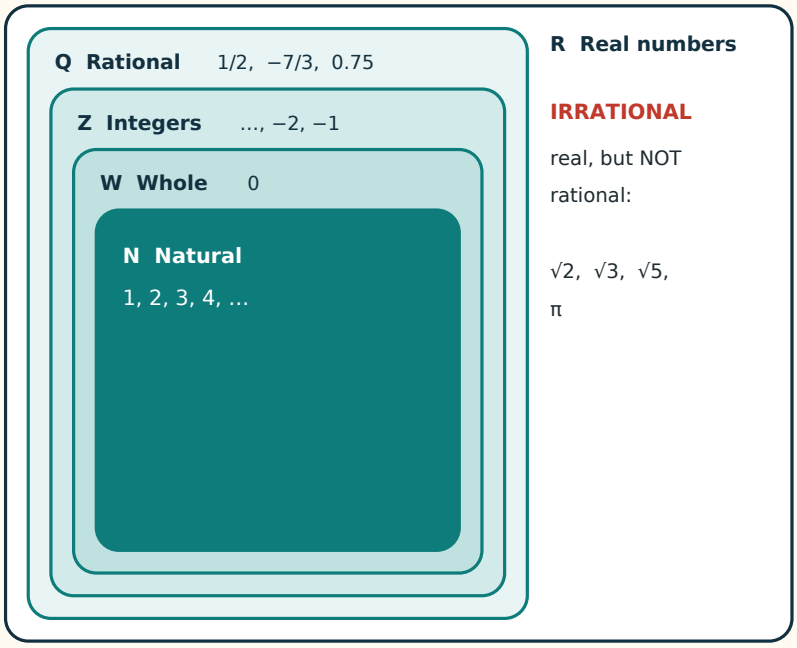
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PREREQUISITE

The number families

Learning objective: Place any number in its families and tell a rational number from an irrational one.

Numbers live in nested families. Each bigger family contains all the smaller ones.



N: counting numbers 1, 2, 3, ...
W: N together with 0
Z: whole numbers and their negatives
Q: numbers that can be written p/q with p, q integers and $q \neq 0$
Irrational: a real number that is not rational
R: all rational and irrational numbers together (every point on the number line)

◆ UNDERSTAND

Every integer is rational: $5 = 5/1$ and $-3 = -3/1$. So a number can sit in several families at once; the *smallest* one tells you the most.

★ SPOT THE CLUE

A root sign does **not** decide it. $\sqrt{9}$ looks irrational but equals 3, so it is a natural number. Always simplify first, then classify.

⚠ COMMON TRAP

“Every square root is irrational.” False: $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{16} = 4$. In Section C you will prove that $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ are irrational. Those are results to be proved, not guesses.

✓ CHECK YOURSELF

C1. Name the *smallest* family (N, W, Z, Q or irrational) for each: -3, 0, $7/2$, $\sqrt{9}$, $\sqrt{7}$, $10/5$. Give a one-line reason for $\sqrt{9}$ and $10/5$.

Why does this matter? A proof that a number is irrational only makes sense once you know exactly what “rational” means. Every such proof starts from the form p/q.

CORE

The Fundamental Theorem of Arithmetic

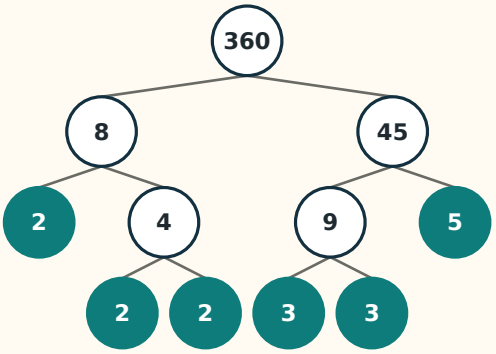
Learning objective: State the theorem, explain what “unique” means, and test it on two different factor trees.

Hold this question: *If I split 360 in different ways, could I ever end up with different primes?*

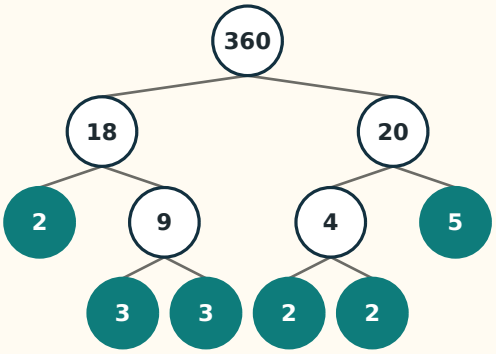
◆ UNDERSTAND

Fundamental Theorem of Arithmetic (FTA). Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
Existence: a prime factorisation always exists. **Uniqueness:** there is only one list of primes, with fixed counts, that multiplies to the number.

Tree A: 360 = 8 × 45 …



Tree B: 360 = 18 × 20 …



Tree A leaves: 2, 2, 2, 3, 3, 5. Tree B leaves: 2, 3, 3, 2, 2, 5. Both give $360 = 2^3 \times 3^2 \times 5$. Check: $8 \times 9 \times 5 = 72 \times 5 = 360$ ✓

? WHY THIS STEP?

The two trees look different, yet the leaves are the same primes in the same amounts. That is what “unique” means. Think of the factorisation as the number’s **fingerprint**: two numbers are equal exactly when their prime fingerprints match.

⚠ COMMON TRAP

“Different trees mean different factorisations.” No: a tree is only a *method*. The FTA is about the primes at the ends, and those never change.

✓ CHECK YOURSELF

C7. Factorise 300 using two different first splits (for example 3 × 100 and 30 × 10). Confirm that both give the same prime factorisation.

✎ YOUR TURN

T1 (transfer). Priya says: “36 = 2 × 18 and 36 = 3 × 12, so 36 has two different factorisations and the FTA is false.” Is she right? Explain.

CORE

Proof that √2 is irrational

Learning objective: Write and explain a complete proof that √2 is irrational, naming the reason for every line.

Hold this question: *Where exactly does the contradiction come from, and why is it a real contradiction?*

➤ CHOOSE THE METHOD

To prove: $\sqrt{2}$ is irrational. Because we must rule out every fraction, we use proof by contradiction, with the Key Fact (p. 15) as the engine.

#	Statement	Reason
1	Assume $\sqrt{2}$ is rational.	We want a contradiction, so assume the opposite of the claim.
2	Then $\sqrt{2} = p/q$ with p, q positive integers, $q \neq 0$, and p, q **coprime**.	Definition of rational; any fraction can be reduced to lowest terms (p. 8). $\sqrt{2} > 0$, so p and q can be positive.
3	Squaring: $2 = p^2/q^2$, so $p^2 = 2q^2$.	Squaring keeps equality; multiply both sides by q^2.
4	So 2 divides p^2.	$p^2 = 2 \times$ (the integer q^2).
5	So 2 divides p.	Key Fact with prime 2 (p. 15).
6	Write $p = 2m$ for some integer m.	Meaning of "2 divides p".
7	Then $(2m)^2 = 2q^2$, so $4m^2 = 2q^2$, so $q^2 = 2m^2$.	Substitute and divide by 2.
8	So 2 divides q^2, hence 2 divides q.	Same reasoning as rows 4 and 5.
9	Now 2 divides p **and** 2 divides q.	Rows 5 and 8.
10	This contradicts "p and q are coprime".	Coprime means 1 is the only common factor; 2 is a common factor greater than 1.
11	So the assumption was false: $\sqrt{2}$ is irrational. ∎	Every other step was valid, so only row 1 can be wrong.

? WHY THIS STEP?

Why is row 10 a true contradiction? Row 2 promised that p and q have *no* common factor above 1. Rows 3 to 9 then produced one (the number 2). The proof defeats its own assumption, not some outside fact.

✓ CHECK YOURSELF

C13. (a) Why are we allowed to assume p and q are coprime in row 2? (b) Which two rows together create the contradiction?

Doubts 1 and 2: theorems and primes

DOUBT 1 *"My teacher says "by the theorem…", but what does a theorem actually tell me? Is it just a rule I have to memorise?"*

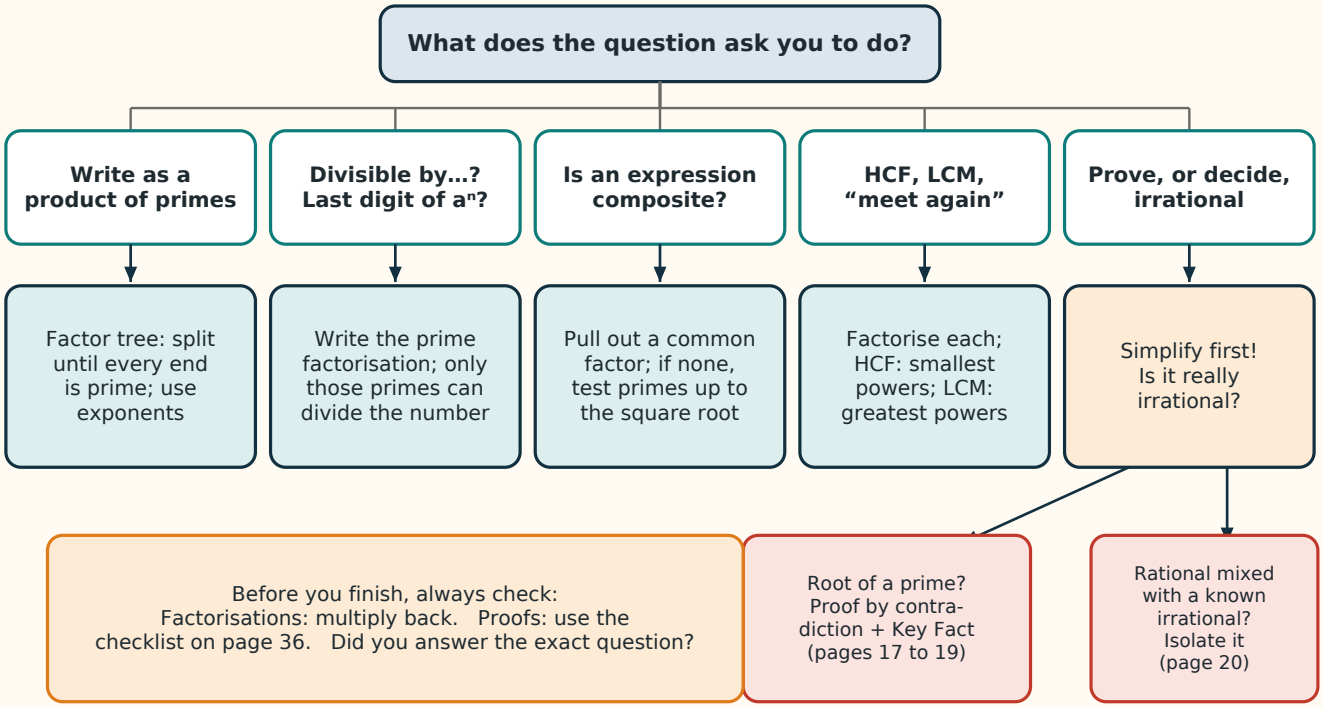
✗ THINKING	A theorem is a rule from the book. I only need to remember the words.
◆ EXPLANATION	A theorem is a statement that has been **proved** from facts already accepted. It has two parts: a condition (the "if") and a conclusion (the "then"). It promises the conclusion *only when the condition holds*.
➤ REASONING	Before using any theorem ask: (1) What must be true first? (2) What does it then promise? If (1) fails, the theorem says nothing.
★ MINI-EXAMPLE	**Key Fact:** *if* p is prime and p divides a^2, *then* p divides a. With p = 3 and $a^2 = 36$ the condition holds (3 divides 36), so 3 divides a = 6 ✓. With p = 4, "p is prime" fails, so nothing is promised; indeed 4 divides 36 but not 6.
✎ YOUR TURN DB1	Write the "if" part and the "then" part of the Key Fact. Then decide: is "if p divides a, then p divides a^2" the same statement? Is it true?
✓ REMEMBER	A theorem is a condition plus a conclusion. Check the condition first.

DOUBT 2 *"Why the fuss about prime factorisation? I can split 12 as 3 × 4 or as 2 × 6. Why must the factors be primes?"*

✗ THINKING	Any split into factors is as good as any other, so the answer should not matter.
◆ EXPLANATION	Splits into *any* factors are not unique: 12 = 3 × 4 = 2 × 6 = 1 × 12. Primes cannot be split further, and the FTA says the primes you end with are always the same. So the prime factorisation is a fixed fingerprint of the number.
➤ REASONING	To compare numbers or decide what divides them, you need something that cannot change. Only the prime factorisation has that property.
★ MINI-EXAMPLE	12 = 3 × 4 and 12 = 2 × 6 look different. Split fully: 3 × (2 × 2) and 2 × (2 × 3). Both become $2^2 \times 3$. Check: 4 × 3 = 12 ✓.
✎ YOUR TURN DB2	Write 30 as a product of two numbers in three different ways. Then give its prime factorisation.
✓ REMEMBER	Many factorisations, but only one prime factorisation.

Decision tree: from question to first step

Learning objective: Follow the branches to choose a method, and know which check to run before you finish.



⚠ COMMON TRAP

A decision tree is a guide, not a machine. A question can mix ideas: for example “show that 6^n cannot end in 0” starts as an **a^n question** but is really about which primes are present. Always inspect the structure.

? WHY THIS STEP?

Every branch ends in an action you already know. Choosing the branch is the real skill: that is why page 32 asks you to name the method *before* solving.

✓ CHECK YOURSELF

Try the tree now. Which branch would you take for “Is √16 irrational?” (Hint: look at the bottom-right box first.)

Mistake clinic (1 of 3)

MISTAKE 1 · Evidence is not proof

✗ STUDENT'S WORK	*Prove that √3 is irrational.* "√3 = 1.7320508... The digits never end and never seem to repeat, so √3 is irrational."
⚠ WHERE IT FAILS	Nothing shows that **no** fraction p/q equals √3. Only finitely many digits can ever be seen.
? WHY TEMPTING	A long, patternless decimal *feels* convincing, and the calculator makes it look like data.
✓ CORRECTED	Assume √3 = p/q with p, q coprime integers, q ≠ 0. Then $p^2 = 3q^2$, so 3 divides p, and so on, ending with 3 dividing both p and q, which contradicts "coprime" (page 18).
➤ PREVENTION RULE	A decimal can suggest a result. Only a logical argument proves it.
✎ FIX IT E1	Explain why "I tried n = 1, 2, 3, 4 and 14^n never ended in 0, so it never does" is not a proof. What argument is needed?

MISTAKE 2 · Stopping before every factor is prime

✗ STUDENT'S WORK	*Find the prime factorisation of 180.* "180 = 2 × 9 × 10."
⚠ WHERE IT FAILS	9 = 3 × 3 and 10 = 2 × 5 are composite. The product equals 180, but the factorisation is unfinished.
? WHY TEMPTING	The multiplication checks (2 × 9 × 10 = 180), so it feels finished.
✓ CORRECTED	180 = 2 × (3 × 3) × (2 × 5) = $2^2 \times 3^2 \times 5$. Check: 4 × 9 × 5 = 180 ✓.
➤ PREVENTION RULE	Scan every factor and ask: is it prime?
✎ FIX IT E2	A student writes 144 = 4 × 36 as "the prime factorisation of 144". Correct it and check your answer.

MISTAKE 3 · Using the Key Fact for a non-prime

✗ STUDENT'S WORK	"$36 = 6^2$ is divisible by 4, so 4 divides 6."
⚠ WHERE IT FAILS	The Key Fact needs a **prime** divisor. $4 = 2^2$ is not prime, and 4 does not divide 6.
? WHY TEMPTING	It copies the pattern "p divides a^2, so p divides a" that works for primes such as 2 and 3.
✓ CORRECTED	4 divides 36 (36 = 4 × 9), but 6 = 2 × 3 has only one 2. What is true: the prime 2 divides 36, so 2 divides 6.
➤ PREVENTION RULE	Before using the Key Fact, check the word "prime".
✎ FIX IT E3	A student says: "9 divides 12^2, so 9 divides 12." Is that right? Which true statement about 3 and 12 can you make instead?

Practice 1: foundation

Learning objective: Check the prerequisite ideas you will rely on later.

1 Guided | 2 Part-guided | 3 Independent | 4 Unfamiliar

Answers: Section H.

P1 Name the *smallest* family (N, W, Z, Q or irrational) for each number: 0, -5, $3/8$, $\sqrt{25}$, $\sqrt{3}$, $12/4$. (You may use that $\sqrt{3}$ is irrational, page 18.)

Purpose: Classify numbers; simplify before deciding.

easy ★☆☆

Working space (use separate paper if needed)

P2 Decide whether each number is prime or composite, with a reason: **57, 61, 87, 101**.

Purpose: Test primality using primes up to the square root.

easy ★☆☆

Working space (use separate paper if needed)

P3 Find the prime factorisation of 420 using a factor tree. Check by multiplying back.

Purpose: Complete a factorisation and verify it.

easy ★☆☆

Working space (use separate paper if needed)

P4 Write $84/126$ in lowest terms. Show the common factor you divided by.

Purpose: Reduce to lowest terms, as every irrationality proof assumes.

easy ★☆☆

Working space (use separate paper if needed)

P5 Are 35 and 48 coprime? Justify using prime factorisation.

Purpose: Apply the definition of coprime.

easy ★☆☆

Working space (use separate paper if needed)

P6 True or false, with a reason: **(a)** Every odd number greater than 1 is prime. **(b)** 2 is the only even prime number.

Purpose: Spot over-generalisations.

moderate ★★☆

Working space (use separate paper if needed)

Revision map

Learning objective: Recall every key statement, the proof structure and the final error checklist on one page.

◆ UNDERSTAND

FTA. Every composite number can be written as a product of primes, and this factorisation is unique apart from the order of the factors.
Key Fact. If a prime p divides a^2, then p divides a (it needs p to be prime).
Rational: can be written p/q, p, q integers, $q \neq 0$. **Coprime:** only common positive factor is 1.
Irrational: real but not rational.



➤ CHOOSE THE METHOD

Pattern 1: √n for a prime n. Assume $\sqrt{n} = p/q$ (coprime). Then $p^2 = nq^2$, so n divides p^2, so n divides p (Key Fact). Write $p = nm$. Then $q^2 = nm^2$, so n divides q. Common factor n contradicts coprime.
Pattern 2: rational combined with a known irrational. Assume the number equals a/b. Isolate the known irrational. It becomes a ratio of integers: rational. This contradicts what is known.

Method quick map

Write as a product of primes	Factor tree, until every end is prime, then multiply back
Divisible by? Last digit of a^n?	Prime factorisation of a^n, then uniqueness: only those primes
Is an expression composite?	Pull out a common factor; otherwise test primes up to the square root
HCF / LCM (enrichment)	Smallest powers for HCF, greatest powers for LCM
Prove √prime is irrational	Pattern 1
Prove a + b√n irrational	Pattern 2

⚠ COMMON TRAP

Final error checklist. ☐ Evidence is not proof (E1) ☐ Every factor prime (E2) ☐ Key Fact only for primes (E3) ☐ A reason for every "so" (E4) ☐ Symbols defined (E5) ☐ Coprime stated (E6) ☐ Squared the coefficient too (E7) ☐ Contradiction named (E8) ☐ Conclusion written, exact question answered (E9)

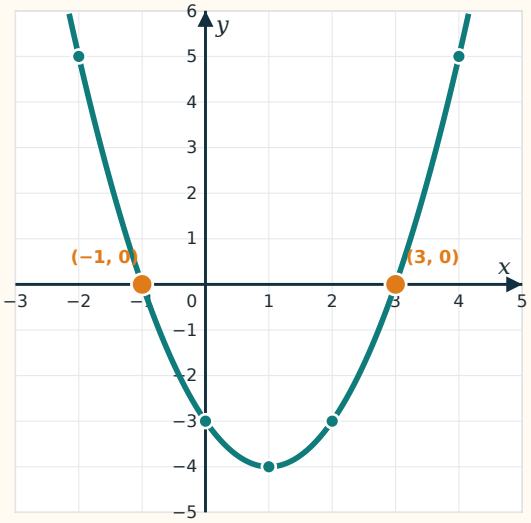
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Polynomials

Understand the Graph.
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A step-by-step learning and practice workbook for Class 10 Mathematics

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WHY IT WORKS

a = 1, b = −2

c = −3

−b/a = 2

c/a = −3

Section C explains why.

$p(x) = x^2 - 2x - 3$

zeroes −1 and 3 · sum 2 · product −3

Core topics: zeroes of a polynomial · zeroes and graphs · zeroes and coefficients of a quadratic

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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PREREQUISITE

Evaluating a polynomial: the value p(k)

Learning objective: Substitute a number into a polynomial correctly, keeping every sign.

◆ UNDERSTAND

p(x) is a rule: put in a number, get out a number. **p(2)** means “replace every x by 2 and work out the result”. The answer is a **value**: a single number.

➤ CHOOSE THE METHOD

To find p(k): (1) write p(x); (2) replace each x by k **inside brackets**; (3) work out powers first; (4) then multiplications; (5) then add and subtract from left to right.

Example. $p(x) = x^2 - 5x + 6$, find $p(-2)$.
$p(-2) = (-2)^2 - 5(-2) + 6 = 4 + 10 + 6 = 20$. **$p(-2) = 20$.**

More values of $p(x) = 2x^2 - 3x + 1$:

x	Substitute with brackets	Value p(x)
2	$2(2)^2 - 3(2) + 1 = 8 - 6 + 1$	**3**
−1	$2(-1)^2 - 3(-1) + 1 = 2 + 3 + 1$	**6**
0	$2(0)^2 - 3(0) + 1 = 0 - 0 + 1$	**1**
½	$2(½)^2 - 3(½) + 1 = ½ - 3/2 + 1$	**0**

Look at the last row: the value is 0. That is special. We meet it on the next page.

? WHY THIS STEP?

Why brackets? $(-2)^2$ means $(-2) \times (-2) = 4$. Without brackets, -2^2 means $-(2 \times 2) = -4$. Brackets keep the sign *inside* the power. They also stop you losing the sign in $5(-2) = -10$, so $-5(-2) = +10$.

⚠ COMMON TRAP

Sign check. A student writes $p(-2) = -2^2 - 5(-2) + 6 = -4 + 10 + 6 = 12$. The first term is wrong: $(-2)^2 = 4$, not -4. The correct value is 20 (see above). Always bracket a negative input.

✓ CHECK YOURSELF

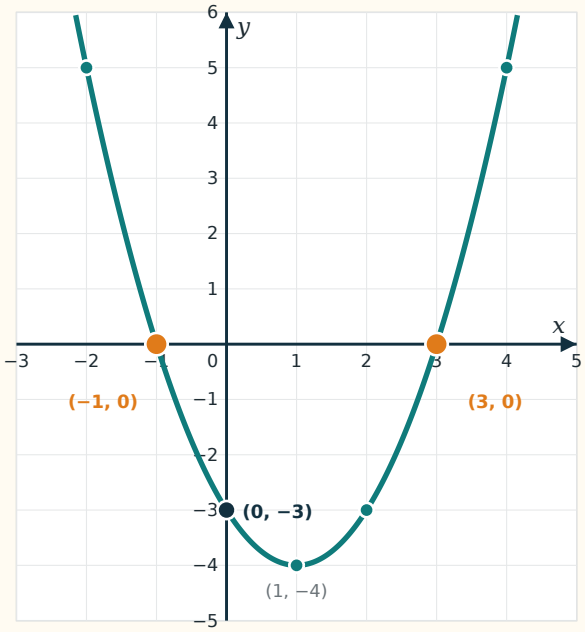
C2. (a) For $p(x) = x^2 + 4x - 2$, find $p(-3)$. (b) For $q(x) = 3x^2 - 2x$, find $q(0)$ and $q(⅓)$.

CORE

The graph connection

Learning objective: Explain why the zeroes of p(x) are the x-coordinates where the graph of y = p(x) meets the x-axis.

Hold this question: *Why would a point on the x-axis have anything to do with a zero?*



y = p(x) = x² − 2x − 3

x	p(x)	Point
−2	**5**	(−2, 5)
−1	**0**	(−1, 0)
0	**−3**	(0, −3)
1	**−4**	(1, −4)
2	**−3**	(2, −3)
3	**0**	(3, 0)
4	**5**	(4, 5)

Orange rows: p(x) = 0, so the point is on the x-axis. Each p(x) was found by substituting, for example p(3) = 9 − 6 − 3 = 0.

◆ UNDERSTAND

The graph of y = p(x) is made of the points (x, p(x)). A point is on the x-axis when its y-value is 0. At such a point p(x) = 0, so its **x-coordinate is a zero**. The graph meets the x-axis at (−1, 0) and (3, 0), so the zeroes are **−1 and 3**. Algebra agrees: $x^2 - 2x - 3 = (x - 3)(x + 1)$.

? WHY THIS STEP?

The x-axis is the line y = 0. A point on both the curve y = p(x) and the line y = 0 must satisfy p(x) = 0. That is why the graph meets the x-axis exactly at the zeroes.

★ SPOT THE CLUE

A zero is the **first number** (x-coordinate) of a point on the x-axis. The point (0, −3), where the graph meets the *y*-axis, shows p(0) = −3. That is a value, not a zero.

⚠ COMMON TRAP

Trusting a rough sketch for exact zeroes. A sketch can suggest where zeroes are. Here we are sure about −1 and 3 because p(−1) = 0 and p(3) = 0 were checked by substitution, and the axes are labelled.

✓ CHECK YOURSELF

C5. (a) Which point gives p(0), and what is p(0)? (b) The lowest point of the graph has y = −4. Why is −4 not a zero?

CORE

Sum and product of zeroes

Learning objective: Find the sum and product of the actual zeroes and compare them with the coefficients a, b and c.

Hold this question: *What links the zeroes 2 and 4 of $x^2 - 6x + 8$ to the numbers 1, −6 and 8?*

ZEROES AND COEFFICIENTS: ONE EXAMPLE

$$2x^2 - 5x + 2$$

a = 2 b = −5 c = 2

zeroes: ½ and 2

since $2x^2 - 5x + 2 = (2x - 1)(x - 2)$

SUM

½ + 2 = 5/2 and −b/a = −(−5)/2 = 5/2 ✓

PRODUCT

½ × 2 = 1 and c/a = 2/2 = 1 ✓

Same numbers, found in two completely different ways.

Polynomial	a, b, c	Zeroes	Sum	−b/a	Product	c/a
$x^2 - 6x + 8$	1, −6, 8	2, 4	6	6	8	8
$2x^2 - 5x + 2$	2, −5, 2	½, 2	5/2	5/2	1	1
$3x^2 + 10x - 8$	3, 10, −8	2/3, −4	−10/3	−10/3	−8/3	−8/3

◆ UNDERSTAND

For $ax^2 + bx + c$ with zeroes α and β: **sum α + β = −b/a** and **product αβ = c/a**. In words: sum = −(coefficient of x) ÷ (coefficient of x^2); product = (constant term) ÷ (coefficient of x^2). Page 16 explains *why*.

➤ CHOOSE THE METHOD

To check: (1) write a, b, c with their signs; (2) find the zeroes; (3) add and multiply them; (4) work out −b/a and c/a; (5) compare and write a conclusion.

⚠ COMMON TRAP

Sign check. In the last row b = +10, but the sum is **−10/3**. The minus sign in −b/a is real: write −b first, then divide by a.

✓ CHECK YOURSELF

C13. Find the zeroes of $5x^2 + 7x - 6$. Add and multiply them, then compare with −b/a and c/a.

✎ YOUR TURN

T5 (transfer). For $4x^2 - 4x - 15$, say what the sum and product of the zeroes *must* be *before* you factorise. Then factorise to confirm.

Doubts 1 and 2: what a zero is

DOUBT 1 *"My teacher says the zero of $x - 3$ is 3. But zero should mean 0! What does "zero" mean here?"*

✗ THINKING A zero of a polynomial is the number 0.

◆ EXPLANATION "A zero of p(x)" is any number k that makes the **value** p(k) equal to 0. The word describes the output, not the number k itself.

▦ SEE IT

x	−1	0	1	2	3	4
p(x) = x − 3	−4	−3	−2	−1	0	1

➤ REASONING Ask: which input gives output 0? That input is the zero.

★ MINI-EXAMPLE $p(x) = x - 3$. $p(3) = 0$, so 3 is the zero. $p(0) = -3$, which is not 0, so 0 is *not* a zero of this polynomial.

✎ YOUR TURN DB1 **DB1** Find the zero of $5x - 20$. Is 0 a zero of it? Give a reason.

✓ REMEMBER A zero is an input that gives output 0.

DOUBT 2 *"Why do I write p(x) = 0 to find zeroes? Why not some other equation?"*

✗ THINKING Writing p(x) = 0 is just a rule to memorise.

◆ EXPLANATION By definition a zero is a number where the value is 0, so finding zeroes means solving p(x) = 0. Setting p(x) equal to another number answers a different question.

▦ SEE IT

x	−4	−3	0	3	4
$p(x) = x^2 - 9$	7	0	−9	0	7

➤ REASONING Match the equation to the question: "value is 0" means p(x) = 0.

★ MINI-EXAMPLE $p(x) = x^2 - 9$. $p(x) = 0$ gives $x^2 = 9$, so $x = \pm 3$: these are the zeroes. $p(x) = 7$ gives $x^2 = 16$, so $x = \pm 4$, but $p(4) = 7$, not 0: so 4 is *not* a zero.

✎ YOUR TURN DB2 **DB2** For $p(x) = x^2 - 9$, solve p(x) = 0 and p(x) = 16. Which equation gives the zeroes?

✓ REMEMBER Zeroes come from p(x) = 0, nothing else.

Decision tree: from question to first step

Learning objective: Follow the branches to choose a method, and know which check to run before you finish.

What does the question give or ask?

- **A graph is given**
  → count and read the x-coordinates of points on the x-axis (pp. 11–12)
- **A number is given: is it a zero? Or find k from a zero**
  → substitute: p(k) = 0 (p. 13)
- **The zeroes are asked**
  → factorise by pattern, set each factor to 0 (p. 14)
- **Sum, product, α² + β², or the zeroes are described**
  → write a, b, c with signs; use −b/a and c/a (pp. 15–17, 19)
- **Build a quadratic from zeroes or from S and P**
  → k[x² − Sx + P] (p. 18)
- **Verify the relationship**
  → find the zeroes, then compare with −b/a and c/a (p. 15)

➤ CHOOSE THE METHOD

Before you finish, always check: substitute each zero you found back into p(x); re-do the sign of every −b/a; make sure the final line answers the exact question asked.

⚠ COMMON TRAP

A decision tree is a guide, not a machine. A question can mix ideas: “If the sum of the zeroes of $kx^2 - 12x + 5$ is 6, find k” needs the relationship (sum = 12/k = 6, so k = 2) and then a check that real zeroes exist.

? WHY THIS STEP?

Every branch ends in an action you already know. Choosing the branch is the real skill, which is why page 32 asks you to name the method *before* solving.

✓ CHECK YOURSELF

Try the tree now. Which branch would you take for “Find the zeroes of $x^2 - 3x$ and verify the relationship”? (More than one branch may be needed.)

Mistake clinic (1 of 3)

MISTAKE 1 — A value is not a zero

✗ STUDENT'S WORK	Is 4 a zero of $p(x) = x^2 - 3x - 2$? "$p(4) = 16 - 12 - 2 = 2$, so 2 is the zero."
⚠ WHERE IT FAILS	2 is the *value* $p(4)$. Because the value is not 0, 4 is not a zero. (2 is not a zero either: $p(2) = -4$.)
? WHY TEMPTING	The number at the end of the calculation feels like "the answer".
✓ CORRECTED	$p(4) = 16 - 12 - 2 = 2 \neq 0$, so 4 is **not** a zero of p.
➤ PREVENTION RULE	Only the output 0 means "zero". Any other output is just a value.
✎ FIX IT E1	**E1** A student finds $p(1) = -2$ for $p(x) = x^2 - 4x + 1$ and writes "so -2 is a zero". Correct it.

MISTAKE 2 — A negative input without brackets

✗ STUDENT'S WORK	Find $p(-2)$ for $p(x) = x^2 - 3x + 1$. "$p(-2) = -2^2 - 3(-2) + 1 = -4 + 6 + 1 = 3$"
⚠ WHERE IT FAILS	-2^2 means $-(2 \times 2) = -4$, but the input was -2, so the first term is $(-2)^2 = +4$.
? WHY TEMPTING	Writing the minus sign next to the 2 looks the same as the sign of the number.
✓ CORRECTED	$p(-2) = (-2)^2 - 3(-2) + 1 = 4 + 6 + 1 = 11$.
➤ PREVENTION RULE	Put every negative input in brackets before you do anything else.
✎ FIX IT E2	**E2** A student writes $p(-3)$ for $p(x) = x^2 + 2x - 4$ as $-3^2 + 2(-3) - 4 = -19$. Correct it.

MISTAKE 3 — A rough sketch treated as exact

✗ STUDENT'S WORK	A sketch of $y = x^2 - 2x - 1$ seems to meet the axis near 2.4 and -0.4. "So the zeroes are exactly 2.4 and -0.4."
⚠ WHERE IT FAILS	$p(2.4) = 5.76 - 4.8 - 1 = -0.04 \neq 0$ and $p(-0.4) = 0.16 + 0.8 - 1 = -0.04 \neq 0$. A sketch gives only an approximation.
? WHY TEMPTING	The graph looks as if it passes through 2.4, and a decimal feels exact.
✓ CORRECTED	The sketch only suggests where the zeroes are. The exact zeroes are $1 + \sqrt{2}$ and $1 - \sqrt{2}$ (sum 2 and product -1 agree with $-b/a$ and c/a).
➤ PREVENTION RULE	A zero from a graph needs a labelled point or $p(k) = 0$ by substitution.
✎ FIX IT E3	**E3** A sketch of $y = x^2 - 4x + 1$ seems to meet the x-axis at 0.3 and 3.7. Evaluate $p(0.3)$ and $p(3.7)$. Are these exact zeroes?

Practice 1: foundation

Learning objective: Check the prerequisite ideas and the meaning of a zero.

1 Guided | 2 Part-guided | 3 Independent | 4 Unfamiliar — Answers: Section H.

P1 Write the degree and the name (linear, quadratic or cubic) of each polynomial, and list all its coefficients: (a) $7 - 2x$ (b) $x^3 + 4x$ (c) $3 - x + 5x^2$.

Purpose: Read the anatomy of a polynomial. **Foundation** ★☆☆

Working space (use separate paper if needed)

P2 For $p(x) = 4x^2 + 3x - 5$ find $p(-2)$ and $p(\tfrac{1}{2})$.

Purpose: Substitute with brackets. **Foundation** ★☆☆

Working space (use separate paper if needed)

P3 Is 3 a zero of $x^2 - 2x - 3$? Is -3? Is 1? Give a reason for each.

Purpose: Test zeroes by substitution. **Foundation** ★☆☆

Working space (use separate paper if needed)

P4 Find the zero of each linear polynomial: (a) $6x - 15$ (b) $4 - 3x$ (c) $x/2 + 3$.

Purpose: Solve $ax + b = 0$. **Foundation** ★☆☆

Working space (use separate paper if needed)

P5 In your own words, explain the difference between “the value of p at 3” and “a zero of p”.

Purpose: Separate a value from a zero. **Standard** ★★☆

Working space (use separate paper if needed)

P6 True or false, with a reason: (a) A polynomial of degree 2 has exactly two zeroes. (b) A zero of a polynomial can be the number 0. (c) Every polynomial has at least one zero.

Purpose: Spot over-generalisations. **Standard** ★★☆

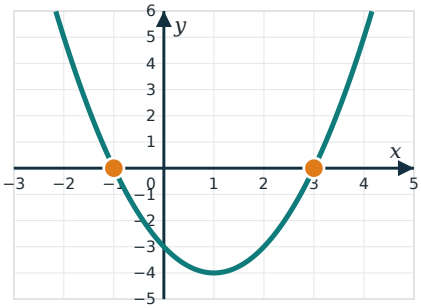
Working space (use separate paper if needed)

Revision map

Learning objective: Recall every key statement, the method clues and the error checklist on one page.

◆ UNDERSTAND

Zero. k is a zero of p(x) if p(k) = 0. Zeroes are the x-coordinates of the points where y = p(x) meets the x-axis (touching counts). A polynomial of degree n has at most n zeroes.
Quadratic $ax^2 + bx + c$ ($a \neq 0$), zeroes α, β: $\alpha + \beta = -b/a$, $\alpha\beta = c/a$. Backwards: $k[x^2 - (\alpha + \beta)x + \alpha\beta]$.
Why: $p(x) = a(x - \alpha)(x - \beta)$ expands to give $b = -a(\alpha + \beta)$ and $c = a\alpha\beta$.



$y = p(x) = x^2 - 2x - 3$

GRAPH ↔ ALGEBRA, ONE PICTURE

zeroes −1 and 3
= x-coordinates where the graph meets the x-axis

sum −1 + 3 = 2
= −b/a = −(−2)/1

product (−1)(3) = −3
= c/a = −3/1

(0, −3) on the y-axis
= the value p(0), not a zero

Question	Method
Is k a zero?	substitute k: p(k) = 0?
Zeroes from a graph	x-coordinates on the x-axis
Find the zeroes	factorise by pattern; solve each factor
Sum, product, $\alpha^2 + \beta^2$	−b/a and c/a, with signs
k from a given zero	put the zero in; set = 0
k from described zeroes	name α, 2α; sum and product
Verify	zeroes first, then compare with −b/a, c/a
Form a quadratic	$k[x^2 - Sx + P]$

⚠ COMMON TRAP

Final error checklist. ☐ a value is not a zero (E1) ☐ bracket negative inputs (E2) ☐ a sketch is not exact (E3) ☐ −b/a has a minus (E4) ☐ c/a, not c (E5) ☐ standard form before a, b, c (E6) ☐ solve each factor = 0 (E7) ☐ common denominators (E8) ☐ compare and conclude (E9)

MATHS WITHOUT GUESSWORK

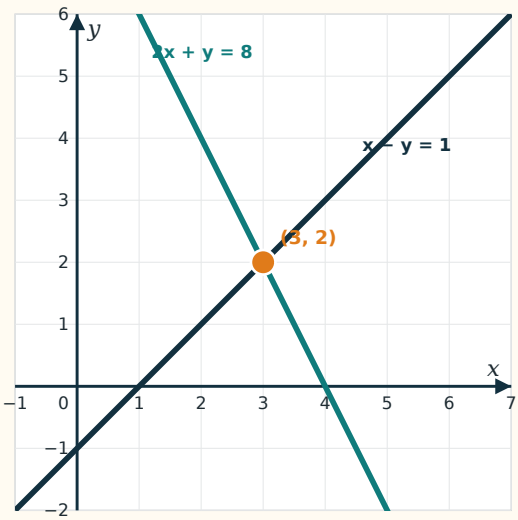
Pair of Linear Equations

in Two Variables

Understand the Relationship.
Choose the Method.
Solve Independently.

A step-by-step concept, problem-solving and board-exam practice workbook for Class 10 Mathematics

CBSE Class 10 Mathematics · Syllabus session 2026–27 · Volume 3



THREE CASES

Intersect
$a_1/a_2 \neq b_1/b_2$

Parallel
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Coincident
$a_1/a_2 = b_1/b_2 = c_1/c_2$

$2x + y = 8$ and $x - y = 1$
meet at one point: the common solution $x = 3, y = 2$

Core topics: graphical method · substitution · elimination · number of solutions · word problems

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

Liked this? The full Pair of Linear Equations in Two Variables workbook is ₹99 →

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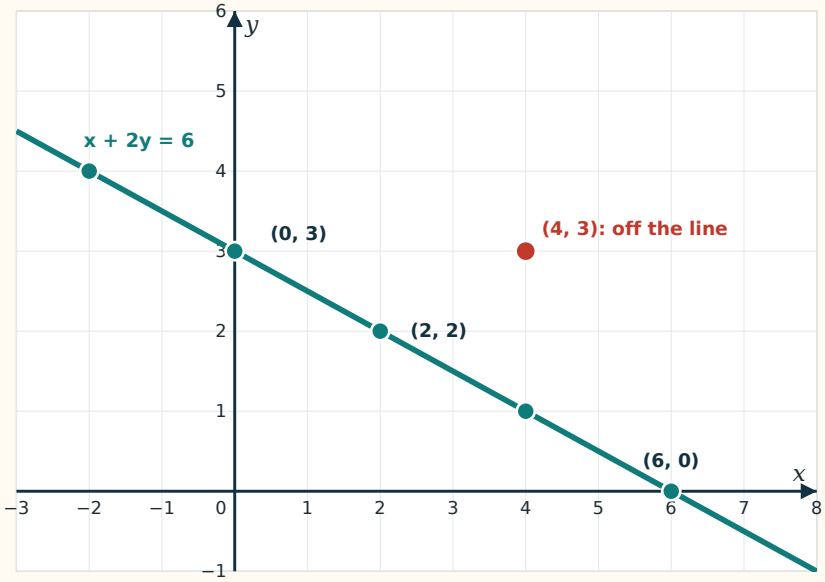
PREREQUISITE

An equation becomes a straight line

Learning objective: Plot solutions of an equation and see that they line up, and that other points do not lie on the line.

Take $x + 2y = 6$. Choose x, find y, and you get a table of solutions. Every solution is a point; plot them and look.

x	6	4	2	0	−2
$y = (6 - x)/2$	0	1	2	3	4



What the picture says

- Every plotted solution lies on one straight line.
- Every other point *on* the line is also a solution (for instance (3, 1.5): $3 + 2(1.5) = 6$).
- A point *off* the line is not a solution: (4, 3) gives $4 + 6 = 10$, not 6.

★ SPOT THE CLUE

Lines with a missing variable. $x = 3$ means x is always 3 and y can be anything, so it is a vertical line through (3, 0). $y = 2$ is a horizontal line through (0, 2). In ratio form they are $1x + 0y - 3 = 0$ and $0x + 1y - 2 = 0$, with a zero coefficient; page 18 returns to this.

➤ CHOOSE THE METHOD

Choosing points to plot. The easiest points make one variable zero: put $x = 0$ to find where the line meets the y-axis (here (0, 3)), and $y = 0$ for the x-axis (here (6, 0)). Plot a **third** point as a safety check: if the three do not lie on one line, one of them is wrong. A straight line needs only two points, but the third catches arithmetic slips.

✎ YOUR TURN

C4 (guided) For $2x - y = 4$: (a) find the points where it meets the axes, (b) find a third point, (c) say whether (3, 3) lies on the line and why.

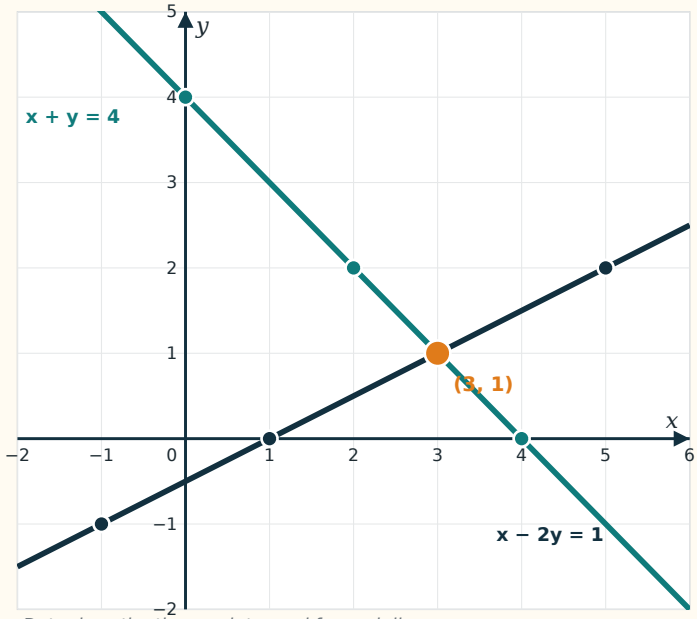
CORE

Graphical method: plot, read, check

Learning objective: Solve a pair by drawing both lines on one graph, reading the crossing point, and confirming it.

SPOT THE PROBLEM. Solve graphically: $x + y = 4$ and $x - 2y = 1$.

Demand: the point that lies on both lines, as an ordered pair (x, y). **Clue:** "graphically" means: draw.



Dots show the three points used for each line.

$x + y = 4$	x	0	4	2
	y	4	0	2
$x - 2y = 1$	x	1	5	−1
	y	0	2	−1

Reading: the lines cross at (3, 1).
Check (1): $3 + 1 = 4$ ✓
Check (2): $3 - 2(1) = 1$ ✓

Answer: x = 3, y = 1. The last column of each table is the third check point; all three points of a line must be in a straight row.

Step	Working	Why this step works
1 Tables	For each equation choose x, find y. Use x = 0 and y = 0 points, plus a third.	Intercepts are quick to compute; the third point catches slips.
2 Plot	Mark each point on one set of axes and join with a straight ruler line, extended both ways.	A line has no end; extending shows where it meets the other line.
3 Read	Find the one point lying on both lines: (3, 1).	A point on both lines satisfies both equations.
4 Check	Substitute into *both* original equations.	The graph can mislead; substitution cannot.

⚠ COMMON TRAP

Plotting an intercept wrongly. For $x - 2y = 1$ at x = 0: $-2y = 1$, so y = −1/2, not −1. Choose x-values that give whole-number y-values, like x = 1, 3, 5 here, so you can plot accurately.

✎ YOUR TURN

C8 (guided) Solve graphically: $x - y = 1$ and $x + y = 7$. Hint: points (1, 0), (0, −1) and (7, 0), (0, 7). Use axes from 0 to 8. State the solution and check it in both equations.

CORE

Elimination: make one unknown vanish

Learning objective: Add or subtract equations so that one variable disappears, and say why this keeps the solution.

? WHY THIS STEP?

Why adding equations is allowed. If two equations are true for a solution, then adding the two left sides gives the same number as adding the two right sides: equals added to equals stay equal. The new equation is also true for the solution. If the y-terms are $+2y$ and $-2y$, they add to 0, so y vanishes and we are left with an equation in x alone.

Example A: opposite coefficients, so add

Solve $3x + 2y = 10$ (1) and $5x - 2y = 22$ (2).

◆ VISUAL 5: ELIMINATION IN ACTION

ORIGINAL	MULTIPLY EVERY TERM	SCALED EQUATION
$3x + \boxed{2y} = 10$	**× 1** →	$3x + \boxed{2y} = 10$
$5x - \boxed{2y} = 22$	**× 1** →	$5x - \boxed{2y} = 22$
ADD (coefficients of y are opposite)		$8x = 32$
		the y-term has become 0

Step	Working	Why this step works
Add	$8x = 32$, so $x = 4$	y-terms $2y$ and $-2y$ cancel.
Back-sub	In (1): $3(4) + 2y = 10$, so $2y = -2$ and $y = -1$	Any original equation works; (1) is simpler.
Verify	(1) $3(4) + 2(-1) = 10$ ✓ (2) $5(4) - 2(-1) = 22$ ✓	Check both originals.

Example B: equal coefficients, so subtract

Solve $4x + 3y = 23$ (1) and $4x - y = 3$ (2).

The x-coefficients are both 4. **Subtract (2) from (1):** $(4x - 4x) + (3y - (-y)) = 23 - 3$, so $4y = 20$ and $y = 5$. Then $4x - 5 = 3$ gives $x = 2$. Verify: $4(2) + 3(5) = 23$ ✓; $4(2) - 5 = 3$ ✓.

⚠ COMMON TRAP

Subtracting a negative. In $3y - (-y)$ the minus-minus gives plus: $3y + y = 4y$. Writing $3y - y = 2y$ is the classic slip. Write the subtraction with brackets around the whole second equation.

✎ YOUR TURN

C12 (guided) Solve $3x + 2y = 7$ and $3x - 4y = -5$ by elimination. Hint: same x-coefficients, so subtract. Give the pair (x, y) and verify.

Doubts 1 and 2: pair, linear, and satisfying both

DOUBT 1 *"Why is it called a pair of LINEAR equations? What is linear about it?"*

✗ THINKING
"Linear" just means the equations are simple or short.

◆ EXPLANATION
An equation is **linear** if each variable has power 1 and no two variables are multiplied. Its graph is a straight line. A **pair** means two such equations taken together about the same x and y.

▦ SEE IT

Equation	Linear in x and y?	Why
$3x - 2y = 7$	yes	x, y to the power 1
$x^2 + y = 3$	no	x^2 is power 2
$xy = 8$	no	x × y: variables multiplied
$y = 5x$	yes	write as $5x - y = 0$

➤ REASONING
Look for powers and products: any x^2, y^2, or xy makes it non-linear.

★ MINI-EXAMPLE
$x + y = 5$ gives $y = 4, 3, 2$ for $x = 1, 2, 3$: points in a straight row. But $x^2 + y = 5$ gives $y = 4, 1, -4$: not a straight row, so not linear.

✎ YOUR TURN DB1
DB1 Which of these are linear in x and y? (a) $3x - 2y = 7$ (b) $xy = 8$ (c) $y = 5x$ (d) $x^2 - y = 1$. Give a reason for each.

✓ REMEMBER
Linear: power 1, no products, straight-line graph.

DOUBT 2 *"My pair (4, 3) works in the first equation, so it is a solution of the pair, right?"*

✗ THINKING
Satisfying one of the two equations is good enough.

◆ EXPLANATION
A solution of the pair must satisfy **both** equations, because the same x and the same y have to obey both conditions. A value that fits only one equation is a solution of that equation alone.

▦ SEE IT

Pair (x, y)	$x + y = 7$	$x - y = 3$	Solution of the pair?
(4, 3)	$4 + 3 = 7$ ✓	$4 - 3 = 1$ ✗	no
(6, 1)	$6 + 1 = 7$ ✓	$6 - 1 = 5$ ✗	no
(5, 2)	$5 + 2 = 7$ ✓	$5 - 2 = 3$ ✓	yes

➤ REASONING
Substitute into equation 1, then *separately* into equation 2. Both must say ✓.

★ MINI-EXAMPLE
For $x + y = 7$ and $x - y = 3$: (4, 3) gives $4 + 3 = 7$ ✓ but $4 - 3 = 1 \neq 3$. So (4, 3) is not a solution of the pair. (5, 2) passes both.

✎ YOUR TURN DB2
DB2 For $2x + y = 9$ and $x - y = 3$, test (3, 3) and (4, 1). Which, if either, is a solution of the pair?

✓ REMEMBER
One check is never enough: test both equations.

Decision tree: from the pair to the first step

Learning objective: Follow the branches from the structure of the pair to a sensible method, and know the check to run last.

VISUAL 7: Look at the pair (or the story)

Q1. Is it a story?
- yes → **Translate first**: define x, y; write two equations. Then continue at Q2 with the equations.
- no ↓

Q2. Does it say “graphically” or “draw”?
- yes → **Graph**: table, plot, read, then substitute to check.
- no ↓

Q3. Does it ask only how many solutions, or the type of lines?
- yes → **Ratios** a_1/a_2, b_1/b_2, c_1/c_2 (zero coefficient: reason directly).
- no ↓

Q4. Is a coefficient 1 or −1, or is a variable already alone?
- yes → **Substitution**: isolate, substitute in the other equation, back-substitute.
- no ↓

Q5. Are the coefficients of one variable equal or opposite?
- yes → **Elimination**: subtract if equal, add if opposite.
- no ↓

Otherwise → **Elimination with multipliers**: multiply every term so one variable matches, then add or subtract.

After any method, always: substitute the values into **both** original equations. If a step ends with only numbers: true (0 = 0) → infinitely many solutions; false (0 = 5) → no solution.

◆ UNDERSTAND

Why this order? Cheap decisions come first. A story must become equations before anything else. Graph and ratio questions are named by their wording. For plain algebra, substitution is cheapest when a variable is easy to isolate; elimination is cheapest when coefficients match; multipliers are the fallback. Different branches may reach the same answer, so use the tree as a guide, not a rule.

✎ YOUR TURN

C17 (guided) Walk each pair through the tree and write the branch you reach: (a) $y = 4x - 3$, $2x + y = 9$ (b) $5x + 2y = 11$, $5x - 3y = 1$ (c) $3x + 5y = 1$, $2x - 3y = 7$ (d) $x - 2y = 4$, $3x - 6y = 7$ (how many solutions?).

Mistake clinic (1 of 3)

MISTAKE 1 — Checking only one equation

✗ STUDENT'S WORK	Is (6, 3) a solution of $x + y = 9$ and $2x - y = 12$? "$6 + 3 = 9$ ✓, so yes."
⚠ WHERE IT FAILS	Only equation (1) was tested. In (2): $2(6) - 3 = 9 \neq 12$, so (6, 3) fails.
? WHY TEMPTING	After one ✓ the job feels finished.
✓ CORRECTED	(6, 3) is **not** a solution of the pair. The actual solution is (7, 2): $7 + 2 = 9$ ✓, $2(7) - 2 = 12$ ✓.
➤ PREVENTION RULE	A pair is a solution only if it makes **both** equations true.
✎ FIX IT E1	**E1** A student says (4, 5) is a solution of $x + y = 9$ and $x - y = 1$ because $4 + 5 = 9$. Correct it.

MISTAKE 2 — Wrong rearranging

✗ STUDENT'S WORK	Write y in terms of x from $3x + 2y = 12$. "$2y = 12 + 3x$, so $y = (12 + 3x)/2$."
⚠ WHERE IT FAILS	Moving 3x to the other side must change its sign: $2y = 12 - 3x$.
? WHY TEMPTING	Moving a term feels like just "sending it across".
✓ CORRECTED	$2y = 12 - 3x$, so $y = (12 - 3x)/2$. Test with (2, 3): $(12 - 6)/2 = 3$ ✓; the wrong version gives $(12 + 6)/2 = 9$ ✗.
➤ PREVENTION RULE	A term changes sign when it crosses "=". Test your rearranged result with a known solution.
✎ FIX IT E2	**E2** A student rearranges $5x - y = 7$ as $y = 5x + 7$. Find the error and correct it.

MISTAKE 3 — Multiplying only one term

✗ STUDENT'S WORK	Solve $x + 2y = 5$, $3x - y = 8$. "Multiply (1) by 3: $3x + 2y = 15$. Subtract (2): $3y = 7$."
⚠ WHERE IT FAILS	Multiplying (1) by 3 gives $3x + 6y = 15$: the 2y must also be multiplied.
? WHY TEMPTING	Only the x was "needed", so only the x gets changed.
✓ CORRECTED	$3x + 6y = 15$; subtract (2): $7y = 7$, $y = 1$, $x = 3$. Check $3 + 2 = 5$ ✓, $9 - 1 = 8$ ✓.
➤ PREVENTION RULE	Multiply every term, both sides.
✎ FIX IT E3	**E3** A student multiplies $2x - 3y = 4$ by 2 and writes $4x - 3y = 8$. Correct it and test with (5, 2).

Practice 1: foundation

Learning objective: Check the prerequisite ideas and the meaning of a solution of an equation and of a pair.

1 Guided | 2 Part-guided | 3 Independent | 4 Unfamiliar

Answers: Section H.

P1 Write each equation in the form $ax + by + c = 0$ and state a, b, c: (a) $2x + 5y = 9$ (b) $y = 3x - 4$ (c) $x/2 - y = 3$.

Purpose: Read coefficients with signs. **Foundation** ★☆☆

Working space (use separate paper if needed)

P2 Which of (1, 4), (2, 1), (−1, 7), (3, −2) are solutions of $3x + y = 7$?

Purpose: Substitute with brackets. **Foundation** ★☆☆

Working space (use separate paper if needed)

P3 Complete a table of solutions of $2x - y = 5$ for x = 0, 1, 3, −1.

Purpose: Find y from x. **Foundation** ★☆☆

Working space (use separate paper if needed)

P4 Is (3, −1) a solution of the pair $x + 2y = 1$ and $3x - y = 10$? Show your check.

Purpose: Test both equations. **Foundation** ★☆☆

Working space (use separate paper if needed)

P5 In your own words, explain why one linear equation in two variables has infinitely many solutions, but a pair usually has just one.

Purpose: Explain the idea of a common solution. **Standard** ★★☆

Working space (use separate paper if needed)

P6 True or false, with a reason: (a) A pair of linear equations always has a solution. (b) The graph of $x + y = 5$ is a straight line. (c) If (2, 3) satisfies one equation of a pair, it is a solution of the pair.

Purpose: Spot over-generalisations. **Standard** ★★☆

Working space (use separate paper if needed)

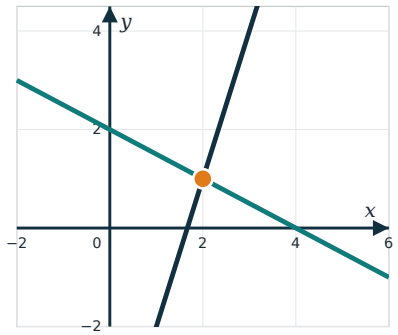
Revision map

Learning objective: Recall the key ideas, the three methods, the ratio conditions and the error checklist on one page.

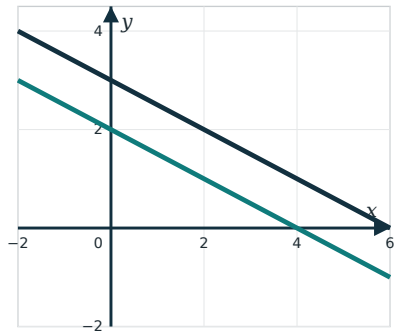
◆ UNDERSTAND

Pair of linear equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$. A solution is an ordered pair (x, y) that satisfies **both**. Each equation is a straight line; the crossing point is the common solution. **Methods:** graph (plot, read, check) · substitution (isolate, substitute in the other equation, back-substitute) · elimination (match a coefficient, add or subtract, back-substitute). **Always** verify in both original equations.

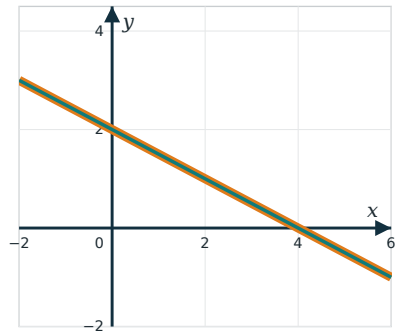
$a_1/a_2 \neq b_1/b_2$
intersecting lines
one solution · consistent



$a_1/a_2 = b_1/b_2 \neq c_1/c_2$
parallel lines
no solution · inconsistent



$a_1/a_2 = b_1/b_2 = c_1/c_2$
coincident lines
infinitely many · dependent (consistent)



Question clue	Method
It is a story	define x, y; two equations; solve; check in the story; answer in words
“Solve graphically”	3-point tables, ruler, read, then substitute to check
A coefficient is 1 or −1	substitution: isolate, bracket, expand, back-substitute
Equal or opposite coefficients	elimination: subtract if equal, add if opposite
Nothing matches	elimination with multipliers: multiply every term
Number of solutions	ratios; zero coefficient: reason directly
Variables vanish	true (0 = 0): infinitely many; false (0 = 5): none

⚠ COMMON TRAP

Final error checklist. ☐ check both equations (E1) ☐ sign when moving a term (E2) ☐ multiply every term (E3) ☐ subtract the whole equation, with brackets (E4) ☐ bracket the substituted expression (E5) ☐ find both variables (E6) ☐ compare all three ratios (E7) ☐ plot points that satisfy the equation (E8) ☐ test word equations with numbers; answer in words (E9)

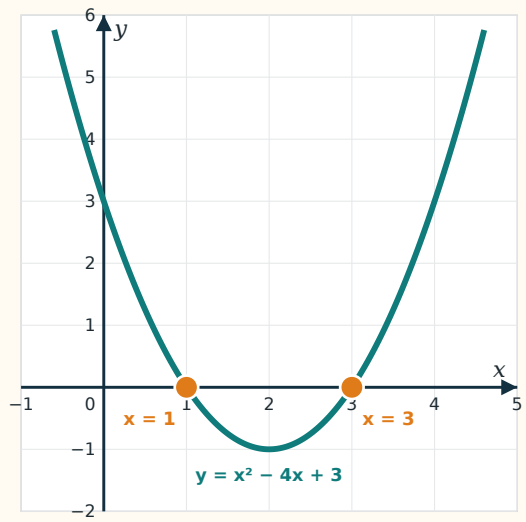
MATHS WITHOUT GUESSWORK

Quadratic Equations

Understand the Equation. Choose the Method. Solve Independently.

A step-by-step concept, problem-solving and board-exam practice workbook for Class 10 Mathematics

CBSE Class 10 Mathematics · Syllabus session 2026–27 · Volume 4



THREE QUESTIONS

1. **What am I** looking at?
2. **What should** I do next?
3. **How do I know** it is correct?

$x^2 - 4x + 3 = 0$

the curve crosses the x-axis at the roots: x = 1 and x = 3

Core topics: standard form · factorisation · quadratic formula · discriminant · word problems

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

Liked this? The full Quadratic Equations workbook is ₹99 →

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1 What am I looking at? | 2 What should I do next? | 3 How do I know it's correct? CORE

What makes an equation quadratic?

Learning objective: Tell an expression from an equation, and name the parts of a quadratic equation.

An **expression** such as $x^2 - 5x + 6$ is a bundle of terms. It has no "=" sign, so there is nothing to solve. An **equation** such as $x^2 - 5x + 6 = 0$ is a statement. It is true for some values of x and false for others. **Solving** means finding the values that make it true.

◆ UNDERSTAND

A **quadratic equation** in x can be written as $ax^2 + bx + c = 0$, where a, b, c are real numbers and **$a \neq 0$**. It has one unknown, and the highest power of that unknown is exactly 2. The word comes from the Latin *quadratus*, "square", because x^2 is a square.

VISUAL 1 · ANATOMY OF A QUADRATIC EQUATION

$$3x^2 - 5x + 2 = 0$$

$3x^2$	$-5x$	$+2$
a = 3	**b = −5**	**c = +2**
coefficient of x^2 (never 0)	coefficient of x (sign included)	constant term (no x at all)

One equation, three disguises. These all describe the same equation and have the same roots (2 and 3). You will often meet the second and third before the first.

Form	Written as	Standard form
Standard	$x^2 - 5x + 6 = 0$	$a = 1, b = -5, c = 6$
Terms on both sides	$x^2 = 5x - 6$	move every term to one side first
Factors in view	$x(x - 5) = -6$	expand, then move: $x^2 - 5x + 6 = 0$

★ SPOT THE CLUE

How a story becomes a quadratic equation. A garden is x metres wide and (x + 3) metres long. Its area is 40 m². Area = length × breadth, so $x(x + 3) = 40$, which becomes $x^2 + 3x - 40 = 0$. Here $a = 1$, $b = 3$, $c = -40$. The clue: a length times a length gave x^2, so the highest power is 2. (Check: $x = 5$ gives $5 \times 8 = 40$ ✓.)

⚠ COMMON TRAP

"It has x^2 in it, so it is quadratic." Not always. $x^2 + 3x = x^2 + 5$ has x^2 on both sides, and they cancel to $3x = 5$, a linear equation. Decide only after you simplify (page 7).

✎ YOUR TURN

C1 (guided) Which of these are quadratic equations? For each one, write a, b and c. (a) $3x^2 - 2x + 1 = 0$ (b) $2x - 5 = 0$ (c) $x^3 + x = 2$ (d) $5 - x^2 = 0$ (e) $y^2 = 16$

1 What am I looking at? 2 What should I do next? 3 How do I know it's correct? CORE

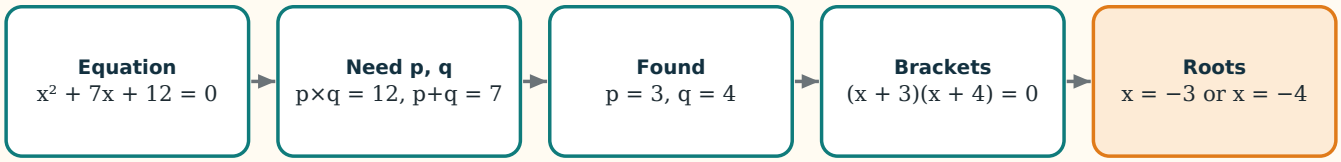
Splitting the middle term: when a = 1

Learning objective: Find the two brackets of $x^2 + bx + c$ using a product-and-sum search.

◆ UNDERSTAND

Why product and sum? Expand $(x + p)(x + q)$: you get $x^2 + (p + q)x + pq$. So to factorise $x^2 + bx + c$ we look for two numbers **p and q with p × q = c and p + q = b**. Then the brackets are $(x + p)(x + q)$.

VISUAL 4 · FACTORISATION WALKTHROUGH



Pairs with product 12 and their sums: 1 + 12 = 13, 2 + 6 = 8, **3 + 4 = 7** ✓. Only the last pair has sum 7.

Worked example (moderate). Solve $x^2 - 2x - 15 = 0$ by splitting the middle term.

Q	Working	Why this step?
① LOOK	$x^2 - 2x - 15 = 0$: quadratic. a = 1, b = −2, c = −15.	Question 1: identify the type; read a, b, c with signs.
② DO	Find two numbers with product ac = 1×(−15) = −15 and sum b = −2: 3 and −5.	Check: 3 + (−5) = −2 ✓; 3 × (−5) = −15 ✓.
	$x^2 + 3x - 5x - 15 = 0$ $x(x + 3) - 5(x + 3) = 0$ $(x - 5)(x + 3) = 0$	Split, group, then take out the common bracket.
	$x - 5 = 0$ or $x + 3 = 0$ so $x = 5$ or $x = -3$.	A product is zero only if a factor is zero.
③ VERIFY	$x = -3$: $(-3)^2 - 2(-3) - 15 = 9 + 6 - 15 = 0$ ✓ $x = 5$: $5^2 - 2 \cdot 5 - 15 = 25 - 10 - 15 = 0$ ✓	Substitute each root into the original equation.

★ SPOT THE CLUE

Sign clue. Look at c first. If **c is positive**, p and q have the *same* sign (both positive if b is positive, both negative if b is negative). If **c is negative**, they have *opposite* signs, and the larger one takes the sign of b. Here c = −15, so one number is positive and one negative; b = −2 means the larger is negative: −5 and +3.

⚠ COMMON TRAP

Right numbers, wrong signs. For $x^2 - 2x - 15$ a student picks 5 and −3 (product −15 ✓, sum +2 ✗). That gives $(x + 5)(x - 3) = x^2 + 2x - 15$, not the original. *Check by expanding:* the sum of the two numbers must equal b **with its sign**. Correct pair: −5 and 3 (sum −2).

✎ YOUR TURN

C8 (part-guided) Solve by factorisation: (a) $x^2 - 9x + 20 = 0$ (b) $x^2 + 3x - 28 = 0$. For each, write the product and sum you need, then check both roots in the original equation.

1 What am I looking at? 2 What should I do next? 3 How do I know it's correct? CORE

The quadratic formula: surd roots and checking

Learning objective: Simplify roots that contain square roots and check them with sum and product.

◆ UNDERSTAND

When D is not a perfect square, the roots contain a square root (a **surd**). They are exact and perfectly correct. Simplify √D by taking out square factors (√20 = √4 × √5 = 2√5), then divide **every** term of the numerator by the common factor.

Example 1. Solve $x^2 - 4x - 1 = 0$.

Q	Working	Why this step?
① LOOK	$x^2 - 4x - 1 = 0$: quadratic, already in standard form. a = 1, b = −4, c = −1.	Read the signs carefully: b is the number with its sign.
② DO	Discriminant: $b^2 - 4ac = (-4)^2 - 4(1)(-1) = 16 - (-4) = 20$. Since D > 0, there are two distinct real roots.	Compute D first, with brackets; it tells you what to expect.
	Substitute: $x = (-(-4) \pm \sqrt{20}) / (2\times1)$	−b = −(−4) = 4; 2a = 2. Brackets stop sign errors.
	$\sqrt{20} = 2\sqrt{5}$ (simplified) $x = (4 \pm 2\sqrt{5}) / 2$ $x = 2 - \sqrt{5}$ or $x = 2 + \sqrt{5}$	Take out the largest square factor, then divide **every** term of the top by the common factor.
③ VERIFY	Sum of roots = 4 = −b/a = 4 ✓; product = −1 = c/a ✓. Decimals: −0.236 and 4.236.	Substitute rational roots; for surd roots use sum = −b/a and product = c/a.

Example 2 (needs cancelling). Solve $3x^2 - 8x + 2 = 0$.

Q	Working	Why this step?
① LOOK	$3x^2 - 8x + 2 = 0$: a = 3, b = −8, c = 2	Standard form; signs read.
② DO	$b^2 - 4ac = (-8)^2 - 4(3)(2) = 64 - 24 = 40$: D = 40 is not a perfect square, so factorising is unlikely.	D tells us roots exist but are not rational.
	$x = (8 \pm \sqrt{40}) / 6 = (8 \pm 2\sqrt{10}) / 6 = (4 \pm \sqrt{10}) / 3$	√40 = 2√10; then divide top and bottom by 2.
③ VERIFY	Sum of roots = 8/3 = −b/a = 8/3 ✓; product = 2/3 = c/a ✓. Decimals: 0.279 and 2.387.	Sum 8/3 = −b/a; product 2/3 = c/a; and decimals look sensible.

⚠ COMMON TRAP

Cancelling only one term. $(4 \pm 2\sqrt{5})/2$ is **not** $4 \pm \sqrt{5}$. The 2 in the denominator must divide both 4 and 2√5: $2 \pm \sqrt{5}$. *Check:* 2 + √5 ≈ 4.236 and $x^2 - 4x - 1$ at 4.236 is ≈ 0 ✓, whereas 4 + √5 ≈ 6.236 gives 38.9 − 24.9 − 1 ≈ 13 ✗.

✎ YOUR TURN

C12 (part-guided) Solve $2x^2 - 6x + 3 = 0$ by the formula. Leave the answer in surd form and check with sum and product.
T2 (transfer) Solve $x^2 + 6x + 4 = 0$ and give each root to 2 decimal places (use √5 ≈ 2.236).

1 What am I looking at? | 2 What should I do next? | 3 How do I know it's correct?

Doubts 1 and 2: type and sign slips

DOUBT 1 *"How do I know whether an equation is quadratic?"* Q1

✗ WHY IT HAPPENS	I look at the equation and decide by eye. If there is an x^2 I say "quadratic", even when other terms hide or cancel it.
◆ SIMPLE EXPLANATION	Simplify the equation completely first. Then look at the **highest power of x** that is left: 2 means quadratic, 1 means linear.
★ SPOT THE CLUE	Brackets multiplying each other, squares or cubes of brackets, and the same power on both sides (which can cancel).
▦ WORKED EXAMPLE	(a) $x^2 + 5x = (x + 1)^2 + 3 \rightarrow x^2 + 5x = x^2 + 2x + 4 \rightarrow 3x - 4 = 0$: **linear**. (b) $(2x - 1)(x + 3) = 6 - x \rightarrow 2x^2 + 5x - 3 = 6 - x \rightarrow 2x^2 + 6x - 9 = 0$: **quadratic**.
? WHY THIS STEP?	Expanding makes hidden x^2 terms appear or cancel. You can only know the degree once everything is on one side and simplified.
⚠ COMMON TRAP	Calling (a) quadratic because it began with x^2. The two x^2 terms cancelled.
✎ YOUR TURN DB1	**DB1** Simplify and decide: (a) $x(x + 3) = x^2 - 4$ (b) $(x - 2)(x + 5) = 2x^2 - 1$ (c) $(x + 2)^3 = x^3 + 8$. Write a, b, c for each quadratic.
✓ QUICK CHECK	After simplifying, the highest power of x tells you the type.

DOUBT 2 *"Terms are on both sides and I keep making sign mistakes when moving them."* Q1

✗ WHY IT HAPPENS	I treat "moving" as a rule to memorise ("change the sign") instead of an action on both sides, so I forget a term or flip the wrong sign.
◆ SIMPLE EXPLANATION	Moving a term means doing the same operation to both sides: to remove $2x^2$ from the right, **subtract $2x^2$** from both sides. The sign changes because you subtracted.
★ SPOT THE CLUE	Quadratic terms or numbers on *both* sides of "=". Aim for the form *expression* = *0*.
▦ WORKED EXAMPLE	$3x^2 - 2x = 2x^2 + 5x - 6$. Subtract $2x^2$: $x^2 - 2x = 5x - 6$. Subtract $5x$: $x^2 - 7x = -6$. Add 6: $x^2 - 7x + 6 = 0$. So $a = 1$, $b = -7$, $c = 6$. Roots: $(x - 1)(x - 6) = 0$, $x = 1$ or 6. *Check $x = 1$ in the original:* $3 - 2 = 1$ and $2 + 5 - 6 = 1$ ✓.
? WHY THIS STEP?	Doing it as three small subtractions or additions keeps every sign tied to an operation.
⚠ COMMON TRAP	Writing $x^2 - 2x + 2x^2 + 5x$ ..., that is, adding instead of subtracting the right side.
✎ YOUR TURN DB2	**DB2** Write in standard form and give a, b, c: (a) $4x^2 + 3 = 5x - 2x^2$ (b) $7 - x^2 = 2(x - 4)$.
✓ QUICK CHECK	Test your standard form with a number: it should hold when the original holds.

1 What am I looking at? | 2 What should I do next? | 3 How do I know it's correct?

CORE

Decision tree: from the equation to the first step

Learning objective: Follow the questions from the top; stop at the first "yes".

VISUAL 8 · METHOD-SELECTION DECISION TREE

Q1. Is it a story in words?
- yes → Define x and form the equation (pp. 19–20); then ask the next questions about that equation.
- no ↓

Q2. Simplify. Is the highest power of x equal to 2?
- no → **No:** it is not quadratic (linear if the highest power is 1). Stop.
- yes ↓

Q3. Is it written as $ax^2 + bx + c = 0$ with a, b, c known?
- no → **No:** expand, move all terms to one side, collect (page 7).
- yes ↓

Q4. Does it ask only for nature of roots, real roots, or a value of k?
- yes → **Yes:** find $D = b^2 - 4ac$ only (pp. 17–18). The roots are not needed.
- no ↓

Q5. Is b = 0 or c = 0?
- yes → **b = 0:** x^2 = number, take $\pm\sqrt{}$. **c = 0:** take x out as a common factor (p. 14).
- no ↓

Q6. Is D a perfect square, or are the numbers small?
- yes → **Yes:** factorise by splitting the middle term (pp. 11–14).

Otherwise (D not a perfect square, messy numbers) → **Use the quadratic formula** (pp. 15–16): a, b, c with brackets, then D, then the roots.

Always, at the end (Question 3): substitute each root (or use sum = −b/a and product = c/a); in a word problem, test each root against the story.

◆ UNDERSTAND

The tree is a guide, not a law. If you can see the factors at once, use them even if the tree says "formula". And a recommended method is never the only valid method: the roots are the same whichever method finds them (see page 31).

✎ YOUR TURN

C18 (guided) Walk each equation through the tree and write the branch: (a) $x^2 - 5x = 0$ (b) $2x^2 - 3x + 7 = 0$ (is it worth solving?) (c) $(x + 2)^2 = 9$ (d) $x^2 - 6x + 7 = 0$

1 What am I looking at? 2 What should I do next? 3 How do I know it's correct?

Mistake clinic (1 of 3)

MISTAKE 1 Rearranging with sign errors

✗ STUDENT'S WORK	Solve $x^2 = 7x - 12$. The student writes $x^2 + 7x - 12 = 0$, so a = 1, b = 7, c = −12.
✗ WHERE IT FAILS	The original has root x = 3 ($9 = 21 - 12$ ✓), but the student's equation gives $9 + 21 - 12 = 18 \neq 0$ ✗. The equation has changed.
? WHY IT TEMPTS	"The terms were already there, so I just copy them across." Moving a term across "=" changes its sign.
✓ CORRECTED	Subtract 7x and add 12 on both sides: $x^2 - 7x + 12 = 0$. So a = 1, b = −7, c = 12. Roots 3 and 4 (check $9 - 21 + 12 = 0$ ✓).
★ PREVENTION RULE	Moving = the same operation on both sides. Test the new equation with a known root.
✎ FIX IT E1	**E1** A student rewrites $2x^2 = 5x - 3$ as $2x^2 + 5x - 3 = 0$. Find the error, correct it and solve.

MISTAKE 2 Misreading a, b, c

✗ STUDENT'S WORK	For $3 - 4x + x^2 = 0$ the student takes a = 3, b = −4, c = 1 and finds roots $(4 \pm 2)/6 = 1, 1/3$.
✗ WHERE IT FAILS	Put x = 1/3 back: $3 - 4/3 + 1/9 \neq 0$ ✗. Only x = 1 works; the other root is wrong because a and c were swapped.
? WHY IT TEMPTS	Reading the numbers from left to right. The first number is not always a.
✓ CORRECTED	a is the coefficient of x^2: $x^2 - 4x + 3 = 0$ gives a = 1, b = −4, c = 3, and roots 1 and 3 (check $9 - 12 + 3 = 0$ ✓).
★ PREVENTION RULE	Rewrite in the order $ax^2 + bx + c = 0$ before reading a, b, c.
✎ FIX IT E2	**E2** Write a, b, c for $4 - 7x + 2x^2 = 0$ and say which value a student would wrongly take as a.

MISTAKE 3 Losing the sign when solving brackets

✗ STUDENT'S WORK	$(2x - 3)(x + 4) = 0$. The student writes x = −3/2 and x = 4 (copying the numbers with their sign).
✗ WHERE IT FAILS	Test x = 4: $(2 \cdot 4 - 3)(4 + 4) = 5 \times 8 = 40 \neq 0$ ✗.
? WHY IT TEMPTS	The bracket shows −3 and +4, so it feels natural to copy them as roots.
✓ CORRECTED	Solve each bracket: $2x - 3 = 0$ gives x = 3/2; $x + 4 = 0$ gives x = −4. (Check x = −4: second bracket is 0 ✓.)
★ PREVENTION RULE	Write "bracket = 0" and solve it; never copy signs.
✎ FIX IT E3	**E3** Find the roots of $(3x + 1)(x - 6) = 0$ and check both.

MISTAKE 4 Wrong factorisation

✗ STUDENT'S WORK	For $2x^2 + 7x + 3 = 0$ the student writes $(2x + 3)(x + 1) = 0$, so x = −3/2 or −1.
✗ WHERE IT FAILS	Expand: $(2x + 3)(x + 1) = 2x^2 + 5x + 3$, not $2x^2 + 7x + 3$. The middle term is wrong.
? WHY IT TEMPTS	Using 2 × 1 for $2x^2$ and 3 × 1 for 3 without checking the middle term.
✓ CORRECTED	ac = 6; numbers with sum 7: 1 and 6. $2x^2 + x + 6x + 3 = (2x + 1)(x + 3)$, so x = −1/2 or −3.
★ PREVENTION RULE	Always expand your factors to see whether you get the original expression.
✎ FIX IT E4	**E4** A student factorises $x^2 + x - 12$ as $(x + 6)(x - 2)$. Check by expanding, then solve correctly.

1 What am I looking at? 2 What should I do next? 3 How do I know it's correct?

Recognise and set up

Learning objective: Question 1 of the ebook: What am I looking at?

1 Guided 2 Part-guided 3 Independent 4 Unfamiliar Guided to part-guided: decide, rewrite, test.

P1 Simplify each equation. Decide whether it is quadratic and, if so, write a, b, c. (a) $2x^2 - 3x + 1 = 0$ (b) $x(x + 3) = x^2 - 5$ (c) $(x + 1)^2 = 2x - 3$ (d) $x^3 - x^2 = 2$

Purpose: Look before you solve **Foundation** ★☆☆

Hint (not the solution): Expand brackets first; compare the highest power.

Working space

P2 Write in the form $ax^2 + bx + c = 0$ with a, b, c. (a) $3x^2 = 5x - 2$ (b) $(x + 2)(x - 3) = 2x$ (c) $5 - x^2 = 4x$ (make a positive)

Purpose: Standard form **Foundation** ★☆☆

Hint (not the solution): Move every term to one side, then collect like terms.

Working space

P3 Is the given number a root? (a) $x = 2$ in $x^2 - 3x + 2 = 0$ (b) $x = -3$ in $x^2 + x - 6 = 0$ (c) $x = 1/2$ in $2x^2 - x - 1 = 0$

Purpose: Test, do not solve **Foundation** ★☆☆

Hint (not the solution): Substitute and see whether the left side becomes 0.

Working space

P4 For what value of k is $(k - 2)x^2 + 3x - 5 = 0$ not a quadratic equation? Then write a, b, c when $k = 5$.

Purpose: Why $a \neq 0$ **Standard** ★★☆

Working space

P5 True or false, with a reason: (a) Every quadratic equation has exactly two real roots. (b) In a quadratic equation a can be 0. (c) If $x = 3$ is a root, then $(x - 3)$ is a factor.

Purpose: Concept check **Standard** ★★☆

Working space

P6 $x = 2$ is a root of $x^2 + kx + 6 = 0$. Find k, then find the other root and verify it.

Purpose: Use a root **Standard** ★★☆

Hint (not the solution): Substitute $x = 2$ to get an equation in k.

Working space

1 What am I looking at? 2 What should I do next? 3 How do I know it's correct?

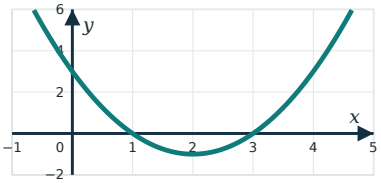
Revision map

Learning objective: Recall the three questions, the methods, the discriminant and the checking habit on one page.

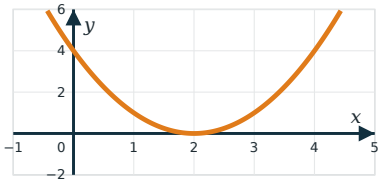
◆ UNDERSTAND

Quadratic equation: $ax^2 + bx + c = 0$, $a \neq 0$, after simplifying. A root is a number that makes the left side 0. It has at most two real roots.
Formula: $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$, with **$D = b^2 - 4ac$**. Put b in brackets; roots come from the formula, D alone is not a root.

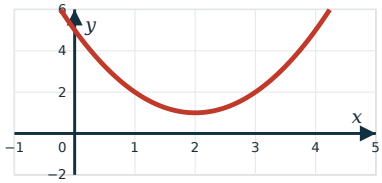
D > 0 · two distinct real roots
cuts x-axis twice



D = 0 · two equal real roots
touches x-axis once



D < 0 · no real roots
misses the x-axis



Question	Your routine
1 What am I looking at?	Simplify; is the highest power 2? Write $ax^2 + bx + c = 0$; read a, b, c with signs.
2 What should I do next?	Special form (c = 0, b = 0) → direct. Nice integer factors → split the middle term. Otherwise → formula. Only the nature asked → D.
3 How do I know it is correct?	Substitute each root (or check sum = −b/a, product = c/a). In a story: reject impossible roots, answer in words with units.

Method	When and how	Code
Special form	$x^2 = k$: $x = \pm\sqrt{k}$ · $ax^2 + bx = 0$: $x(ax + b) = 0$	S
Factorise	Find two numbers with product ac and sum b; split, group, set each bracket to 0	F
Formula	Works for every quadratic; best when D is not a perfect square	Q
Discriminant only	"nature of roots", "is it possible", find k for equal or real roots	D
Root check	"is x = … a root?": substitute, see whether you get 0	R

⚠ COMMON TRAP

Final error checklist. ☐ moved terms with the sign changed (E1) ☐ a, b, c read after standard form (E2) ☐ solved each bracket, not copied signs (E3) ☐ expanded factors to test (E4) ☐ b in brackets in D (E5) ☐ both ± values (E6) ☐ factorised instead of dividing by x (E7) ☐ D is not a root (E8) ☐ D = 0 still has a root (E9) ☐ tested roots against the story (E10)

✓ CHECK YOURSELF

Checking habit. Substitute every root in the *original* equation. For surd roots check the sum and product. Then ask: does the answer make sense for the question?

Progress: Section I ✓ (revision). Next: page 50, record where you stand and plan what to practise.

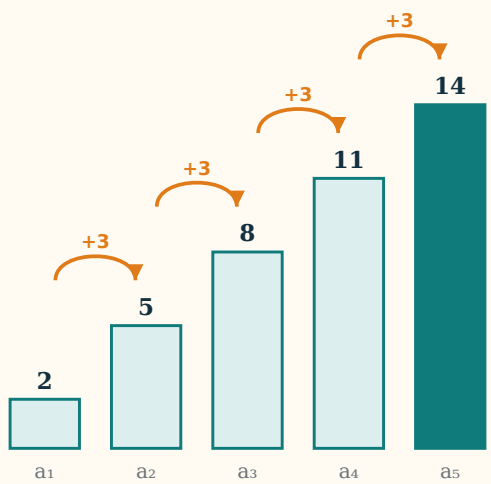
MATHS WITHOUT GUESSWORK

Arithmetic Progressions

Understand the Pattern.
Choose the Formula.
Solve Independently.

A step-by-step concept, problem-solving and board-exam practice workbook for Class 10 Mathematics

CBSE Class 10 Mathematics · Syllabus session 2026–27 · Volume 5



2, 5, 8, 11, 14, …

every step adds the same amount: d = 3

TWO SIGNATURE QUESTIONS

1 **Why does the** formula use (n − 1)?

2 **Nth term or** sum formula?

Core topics: pattern · common difference · nth term · sum of n terms · daily-life problems

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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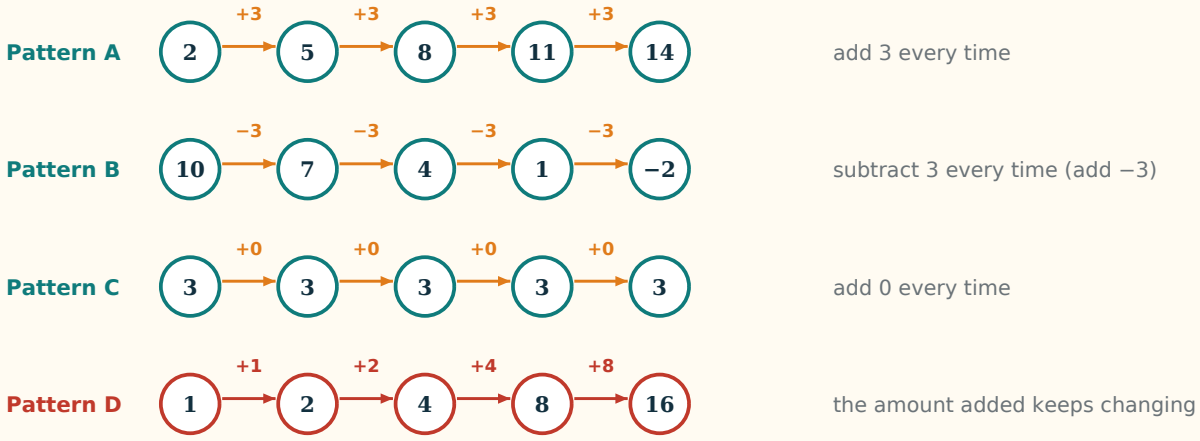
★ Signature 1 · Why (n − 1)? ★ Signature 2 · Term or sum? CORE

Patterns before formulas

Learning objective: See what stays the same in a pattern, and what a “term” is, before any formula appears.

A list of numbers that follows a rule is called a **sequence**; each number in it is a **term**. Look at how each term is made from the one before it.

VISUAL 1 · THE NUMBER-PATTERN VISUAL: WHAT HAPPENS FROM ONE TERM TO THE NEXT?



List	Rule in words	Same amount added each time?
2, 5, 8, 11, 14, …	each term is 3 more than the one before	**Yes**: +3
10, 7, 4, 1, −2, …	each term is 3 less than the one before	**Yes**: −3
3, 3, 3, 3, 3, …	each term equals the one before	**Yes**: +0
1, 2, 4, 8, 16, …	each term is twice the one before (the gaps are 1, 2, 4, 8)	**No**: the gap changes

◆ UNDERSTAND

Arithmetic progression (AP): a list of numbers in which each term, except the first, is obtained by adding the **same fixed number** to the term before it. That fixed number is called the **common difference**, written **d**. Patterns A, B and C are APs; pattern D is not, because the amount added changes (1, 2, 4, 8).

★ SPOT THE CLUE

The key phrase is “the same each time”. A list can grow steadily, shrink steadily or stay put and still be an AP. What matters is that the *amount added* never changes, not that the numbers get bigger.

✎ YOUR TURN

C1 (guided) For each list say what is added (or subtracted) each time, or write “no fixed amount”: (a) 6, 11, 16, 21 (b) 50, 44, 38, 32 (c) −5, −5, −5, −5 (d) 2, 4, 7, 11 (e) 1/2, 1, 3/2, 2

★ Signature 1 · Why (n − 1)? ★ Signature 2 · Term or sum? CORE

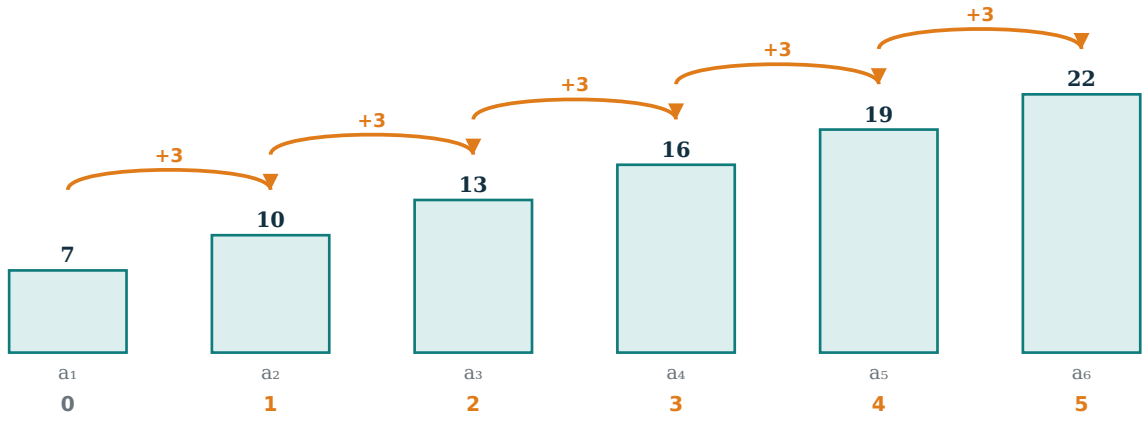
Why does the formula use (n − 1)?

Learning objective: Signature 1, part 1 of 3. Discover why the n-th term needs n − 1 jumps. No memorising yet.

Everyone is told "$a_n = a + (n - 1)d$". Hardly anyone is told **why**. Let us build it from the picture, using the AP 7, 10, 13, 16, 19, 22, ... (a = 7, d = 3).

VISUAL 5 · THE STAIRCASE: TO STAND ON STEP n YOU HAVE JUMPED n − 1 TIMES

You START on the first step. You have not jumped at all yet: 0 jumps.



position n: term name above, number of jumps from a_1 below (orange)

Term	Position n	Jumps from a_1	As an expression	Value
a_1	1	0	a	7
a_2	2	1	a + d	10
a_3	3	2	a + 2d	13
a_4	4	3	a + 3d	16
a_5	5	4	a + 4d	19
a_{10}	10	9	a + 9d	34
a_n	n	n − 1	a + (n − 1)d	?

FENCE POSTS



5 posts, 4 gaps

◆ UNDERSTAND

Read the table downwards. The number of jumps is always **one less** than the position: position 2 needs 1 jump, position 5 needs 4 jumps, position 10 needs 9. Each jump adds d. So reaching position n takes **n − 1** jumps, and the value is **a + (n − 1) × d**. It is the same reason a fence with 5 posts has only 4 gaps.

? WHY THIS STEP?

Why not a + n × d? Because you start *on* the first term. If the first term were called the "0th" term, then a + nd would be right. We number terms from 1, so the first term has taken 0 jumps, the second 1 jump, and so on: always one fewer.

✓ CHECK YOURSELF

Hold this question. How many jumps does it take to reach the 25th term of an AP? And the 100th? (Answers: 24 and 99. Now try W1–W6 on page 14 when you have seen the formula on page 13.)

★ Signature 1 · Why (n − 1)? ★ Signature 2 · Term or sum? CORE

Method 3: using the sum formulas

Learning objective: Use $S_n = n/2[2a + (n - 1)d]$ and $S_n = n/2(a + l)$, see why they agree, and know when each is convenient.

◆ UNDERSTAND

Form 1: $S_n = (n/2)[2a + (n - 1)d]$ (know a, d, n). **Form 2:** $S_n = (n/2)(a + l)$ where **$l = a_n$** is the last term (know a, l, n). They agree because $l = a + (n - 1)d$, so $a + l = 2a + (n - 1)d$: the same pair total.

Same AP: 4, 9, 14, 19, 24, 29, 34, 39 (n = 8)	Form 1: a, d, n	Form 2: a, l, n
Formula	$(n/2)[2a + (n - 1)d]$	$(n/2)(a + l)$
Substitute	$(8/2)[2(4) + 7 \times 5]$	$(8/2)(4 + 39)$
Pair total	$8 + 35 = 43$	$4 + 39 = 43$
Sum	$4 \times 43 = 172$	$4 \times 43 = 172$

Worked example 1 (form 1, negative d). Find the sum of the first 20 terms of 30, 26, 22, ... Predict the sign first.

Step	Working	Why this step?
READ	Asked: S_{20} (a total). Given: $a = 30$, $d = -4$, $n = 20$. *Predict:* terms go negative after the 8th: expect a negative total.	Predicting catches sign slips later.
CHOOSE	$S_n = (n/2)[2a + (n - 1)d]$	a, d and n are known, the last term is not.
SOLVE	$S_{20} = 20/2\ [2(30) + (20 - 1)(-4)] = 10 \times [60 - 76] = 10 \times (-16) = -160$	Put d in brackets: $19 \times (-4) = -76$.
CHECK	Form 2 with $l = a_{20} = -46$: $10 \times (30 - 46) = -160$ ✓, negative as predicted.	Use the other form as a check.

Worked example 2 (form 2, last term given). Find $5 + 8 + 11 + \ldots + 62$.

Step	Working	Why this step?
READ	Asked: the total. Given: $a = 5$, $d = 3$, $l = 62$; n is not given.	The list is given with its last term.
CHOOSE	Find n: $(62 - 5)/3 + 1 = 20$. Then $S_n = (n/2)(a + l)$.	Form 2 uses l directly; no need for d again.
SOLVE	$S_{20} = 20/2\ (a + l) = 20/2\ (5 + 62) = 10 \times 67 = 670$	The pair total is $5 + 62 = 67$.
CHECK	Form 1: $(20/2)[10 + 19 \times 3] = 10 \times 67 = 670$ ✓.	Both forms must match.

⚠ COMMON TRAP

Two slips. (1) Dropping the sign of d: $2(30) + 19 \times 4 = 136$ gives 1360, but S_{20} cannot exceed $20 \times 30 = 600$. (2) Using l as n: $l = 62$ is the last term; $n = 20$.

✎ YOUR TURN

C11 (guided) Find the sum of: (a) the first 15 terms of 6, 10, 14, ... (b) the first 12 terms of −5, −2, 1, ... **C12** Find $7 + 11 + 15 + \ldots + 63$. **T3** (transfer) Sum of all two-digit numbers divisible by 4.

★ Signature 1 · Why (n − 1)? ★ Signature 2 · Term or sum?

Doubts 1 and 2: AP test, constant d

DOUBT 1 *"How do I know whether a list of numbers is an AP?"*

✗ WHY IT HAPPENS	I compare the first two numbers, see a pattern, and assume the rest follow it.
◆ SIMPLE EXPLANATION	Find **every** gap (next − previous). If all gaps are equal the list is an AP; one different gap means it is not.
★ SPOT THE CLUE	"Show that…", "Is this an AP?", or a list of four or more numbers: check all neighbouring pairs.
▦ WORKED EXAMPLE	Is $3, 8, 13, 18, 24$ an AP? Gaps: $8 - 3 = 5$, $13 - 8 = 5$, $18 - 13 = 5$, $24 - 18 = 6$. The last gap is 6, so it is **not** an AP.
? WHY THIS STEP?	A list of k numbers has k − 1 gaps; every one has to match.
⚠ COMMON TRAP	Thinking an AP must go up. $20, 14, 8, 2$ goes down, with $d = -6$, and is a perfectly good AP.
✎ YOUR TURN DB1	**DB1** Which are APs? (a) $2, 7, 12, 17, 23$ (b) $15, 11, 7, 3, -1$ (c) $1/2, 1, 3/2, 2, 5/2$ (d) $4, 4, 4, 4, 5$.
✓ QUICK CHECK	Count the gaps and write them in a row: they should all look the same.

DOUBT 2 *"Why does the difference have to be constant? What goes wrong if it is not?"*

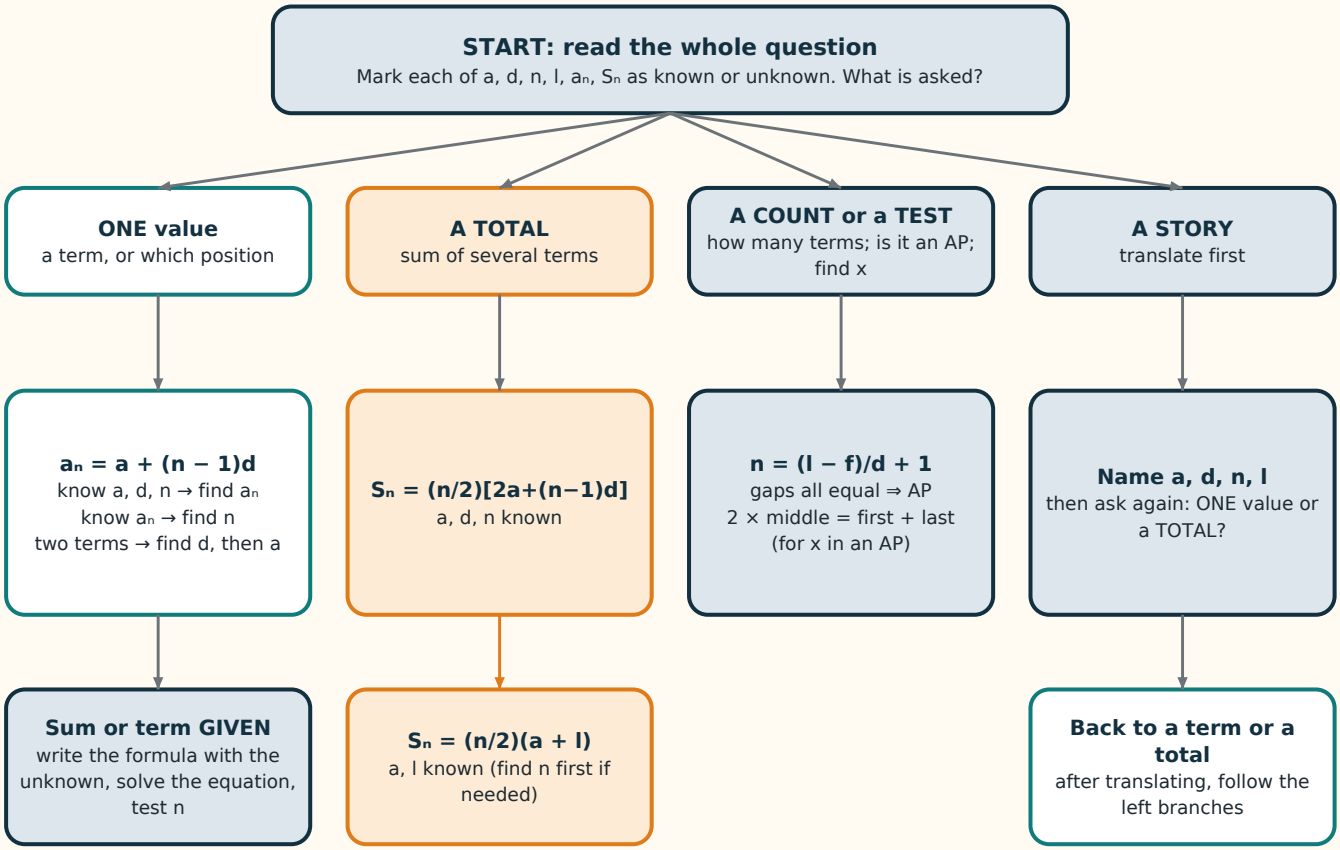
✗ WHY IT HAPPENS	"Constant" sounds like a fussy rule, not a reason.
◆ SIMPLE EXPLANATION	Every formula in this chapter counts **equal jumps**: the n-th term formula adds (n − 1) jumps of size d, and the pairing trick needs equal gaps. If the gaps change, those formulas give wrong answers.
★ SPOT THE CLUE	A question that uses a, d and a formula assumes the list really is an AP.
▦ WORKED EXAMPLE	The list $2, 5, 9, 14$ has gaps 3, 4, 5. Using $a = 2$, $d = 3$ the formula gives $a_4 = 2 + 3 \times 3 = 11$, but the list's 4th term is 14.
? WHY THIS STEP?	The formula assumed every jump was 3, but the real jumps grow.
⚠ COMMON TRAP	Taking d from the first gap only and trusting the formula without testing the other gaps.
✎ YOUR TURN DB2	**DB2** For $1, 4, 9, 16$ use $a = 1$, $d = 3$ to predict the 4th term, compare it with the actual 4th term, and explain the mismatch.
✓ QUICK CHECK	If a formula's predictions do not match the list, the list is not an AP.

★ Signature 1 · Why (n − 1)? ★ Signature 2 · Term or sum? CORE

The formula decision tree

Learning objective: Signature 2, part 2 of 4. Follow the branch that matches what the question asks.

VISUAL 8 · FORMULA-SELECTION DECISION TREE



ALWAYS: n is a positive whole number · check another way · answer in words, with units

Stuck between two branches? Ask: is the answer a single value from the list, or a total of values?

? WHY THIS STEP?

Why the tree starts with the question, not the numbers. The same list of numbers can lead to a term, a total, a position or a count. What the question asks decides the branch; the given numbers decide the form of the formula.

✎ YOUR TURN

C15 (guided) Walk each question through the tree and write the branch: (a) “Find the 14th term of 2, 9, 16, ...” (b) “Find the sum of the first 14 terms of 2, 9, 16, ...” (c) “How many terms has 2, 9, 16, ..., 100?” (d) “Which term of 2, 9, 16, ... is 93?”

★ Signature 1 · Why (n − 1)? ★ Signature 2 · Term or sum? CORE

Mistake clinic (1 of 3)

Learning objective: Read each student's work, find the exact line that fails, then fix the one in the box.

MISTAKE 1 — Subtracting in the wrong order

✗ STUDENT'S WORK	For $20, 17, 14, \ldots$ the student writes $d = 20 - 17 = 3$ and then $a_{10} = 20 + 9 \times 3 = 47$.
✗ WHERE IT FAILS	The list is going *down*, yet the answer 47 is far above 20. The true common difference is $17 - 20 = -3$.
? WHY IT TEMPTS	Subtracting the bigger number from the smaller feels backwards, so the student subtracts the first from the second reversed.
✓ CORRECTED	$d = 17 - 20 = -3$, so $a_{10} = 20 + 9 \times (-3) = 20 - 27 = -7$. A falling list gives a falling tenth term ✓.
★ PREVENTION RULE	d = next term − previous term. Falling list ⇒ d is negative. Check the direction of your answer.
✎ FIX IT MC1	**MC1** A student finds d for $50, 46, 42, \ldots$ as $50 - 46 = 4$. Correct it and find the 8th term.

MISTAKE 2 — Using n instead of (n − 1) SIGNATURE 1

✗ STUDENT'S WORK	Find the 12th term of $4, 9, 14, \ldots$. The student writes $a_{12} = 4 + 12 \times 5 = 64$.
✗ WHERE IT FAILS	Test the student's rule on the 2nd term: $4 + 2 \times 5 = 14$, but the 2nd term is 9 ✗. The rule always overshoots by one d.
? WHY IT TEMPTS	"The 12th term, so multiply by 12." Position and number of jumps feel like the same thing.
✓ CORRECTED	From term 1 to term 12 there are 11 jumps: $a_{12} = 4 + 11 \times 5 = 59$. Test: $a_2 = 4 + 1 \times 5 = 9$ ✓.
★ PREVENTION RULE	The first term has no jump. The nth term has (n − 1) jumps. Test the formula on n = 2.
✎ FIX IT MC2	**MC2** A student writes the 15th term of $6, 10, 14, \ldots$ as $6 + 15 \times 4 = 66$. Find the error, then the true value.

MISTAKE 3 — Finding a term when a sum is asked SIGNATURE 2

✗ STUDENT'S WORK	Find the sum of the first 10 terms of $5, 8, 11, \ldots$. The student finds $a_{10} = 5 + 9 \times 3 = 32$ and writes "sum = 32".
✗ WHERE IT FAILS	The sum has ten terms, each at least 5, so it must be at least 50. 32 is only the *last* term.
? WHY IT TEMPTS	The word "10" and the formula a_n are familiar, so the question is read as "find the 10th term".
✓ CORRECTED	$S_{10} = (10/2)[2(5) + 9 \times 3] = 5 \times 37 = 185$. Check with form 2: $5(5 + 32) = 185$ ✓.
★ PREVENTION RULE	"Sum", "total", "altogether" ⇒ S_n. Ask: is the answer one value or a total?
✎ FIX IT MC3	**MC3** Find the sum of the first 12 terms of $4, 7, 10, \ldots$. (A student wrote 37. Say what that number actually is.)

★ Signature 1 · Why (n − 1)? ★ Signature 2 · Term or sum? PRACTICE

Level 1: recognise an AP and find d

Learning objective: Decide whether a list is an AP, find d with its sign, and write terms from a and d.

1 Recognise | 2 Terms | 3 Sums | 4 Unknowns | 5 Mixed | Test

Take your time. Write d for each pair.

P1 Which of these are APs? Give d for each AP. (a) $4, 9, 14, 19$ (b) $2, 4, 8, 16$ (c) $10, 7, 4, 1$ (d) $1, 1, 1, 1$

Purpose: test the gaps

Hint (not the solution): next − previous, for every pair

Working space

P2 Find d. (a) $13, 19, 25, \ldots$ (b) $5, 1, -3, \ldots$ (c) $0.5, 1.0, 1.5, \ldots$ (d) $-8, -5, -2, \ldots$

Purpose: d with sign

Hint (not the solution): subtract in the same order each time

Working space

P3 Write the first four terms of the AP with $a = 12$ and $d = -5$.

Purpose: build from a and d

Working space

P4 A list has the rule $a_n = 3n + 2$. Write its first three terms and show that it is an AP.

Purpose: rule to list

Hint (not the solution): put n = 1, 2, 3, then check the gaps

Working space

P5 Find d and the next two terms of $2.5, 4, 5.5, \ldots$.

Purpose: decimal d

Working space

P6 Find x so that $x, 2x + 1, 5x - 2$ are in AP. Write the three terms.

Purpose: middle = average

Hint (not the solution): in an AP, 2 × middle = first + last

Working space

★ Signature 1 · Why (n − 1)? ★ Signature 2 · Term or sum? REVISION

Revision map

Learning objective: VISUAL 12. The whole chapter on one page: the pattern, the formulas, how to choose, and what to check.

VISUAL 12 · REVISION MAP

AP
list with the same d each time → **d**
next − previous (with sign) → **$a_n = a + (n - 1)d$**
ONE term; n − 1 jumps → **S_n**
a TOTAL of n terms

Symbol	Meaning	Symbol	Meaning
a	first term	l	last term (a_n for the final term)
d	common difference: $a_{n+1} - a_n$	a_n	the nth term (one value)
n	position or number of terms (a positive whole number)	S_n	sum of the first n terms (a total)

Formula	Use it when	Check
$a_n = a + (n - 1)d$	a term, a position n, d or a from known terms; n − 1 = number of jumps	test n = 1 and n = 2
$S_n = (n/2)[2a + (n - 1)d]$	a, d, n known (a total is asked)	compare with the other form
$S_n = (n/2)(a + l)$	first and last terms known; find n first if missing	estimate: average × n
$n = (l - a)/d + 1$	counting terms in a list or multiples	n must be a whole number
$a_n = S_n - S_{n-1}$	a term is wanted and S_n is given as a formula	$a_1 = S_1$

SIGNATURE 1 · Why (n − 1)?

- First term: 0 jumps. nth term: (n − 1) jumps of d.
- Position 12 means 11 jumps, so $a_{12} = a + 11d$.
- Test the rule on n = 1 and n = 2 (MC2).

SIGNATURE 2 · Term or sum?

- One value or a position → a_n. A total → S_n.
- Name what is asked, what is given, then choose.
- A given term can serve as l in the sum (MC3, MC7).

⚠ COMMON TRAP

Mistake checklist. ☐ d in the right order (MC1) ☐ n − 1, not n (MC2) ☐ term, not total, when one term is asked (MC3, MC7) ☐ a and d not swapped (MC4) ☐ negative d in brackets (MC5) ☐ n and l kept apart (MC6) ☐ arithmetic rechecked (MC8) ☐ n a whole number (MC9) ☐ roots tested against the story (MC10)

✓ CHECK YOURSELF

Checking routine. Re-read the question. Substitute n = 1. Compute the answer a second way (other sum form, or average × n). Does the sign and size make sense? Answer in words with units.

Progress: Section I ✓. Next: page 50, record where you stand and plan your practice.

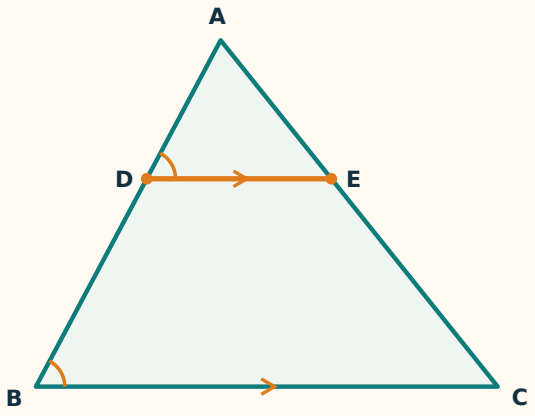
MATHS WITHOUT GUESSWORK

Triangles

Understand the Diagram.
Recognise the Theorem.
Prove with Confidence.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27



Clue: DE ∥ BC
Theorem: Basic Proportionality Theorem
Step: AD/DB = AE/EC

SIGNATURE FEATURE

How do I know which theorem to use, and why is this proof step valid?

Every proof in this book is built from the same three links: the **clue** in the diagram, the **theorem** that matches it, and the **step** it lets you write. Learn the links and you can start a proof you have never seen before.

Core topics: similar figures · Basic Proportionality Theorem and its converse · AA, SSS and SAS similarity criteria

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

Liked this? The full Triangles workbook is ₹99 →

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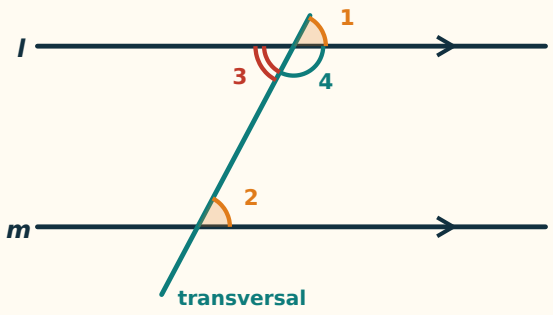
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Angle facts: your bank of reasons

PREREQUISITE

Learning objective: Recall the angle facts that become the reasons in every similarity proof, and know the clue that allows each one.

Hold this question: *Where do the "equal angles" in a proof actually come from?*



l ∥ m. ∠1 and ∠2 are corresponding angles; ∠1 and ∠3 are vertically opposite.

◆ UNDERSTAND

When a **transversal** crosses two **parallel** lines:

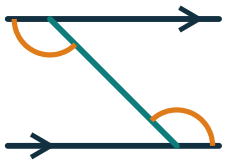
- **corresponding** angles are equal (∠1 = ∠2, same position at each crossing);
- **alternate** angles are equal (on opposite sides of the transversal, between the lines);
- **co-interior** angles add to 180°.

Without the parallel mark, none of these is guaranteed.

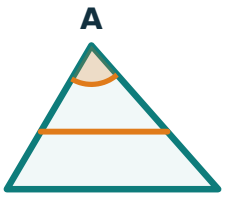
Fact	Clue that allows it	What you write in a proof
Corresponding angles are equal	*two lines marked ∥, cut by a line*	**∠ADE = ∠ABC (corresponding angles, DE ∥ BC)**
Alternate angles are equal	*∥ lines with a crossing line between them (Z-shape)*	**∠OAB = ∠OCD (alternate angles, AB ∥ CD)**
Vertically opposite angles are equal	*two straight lines crossing (X-shape)*	**∠AOB = ∠COD (vertically opposite angles)**
Common angle	*two triangles share a vertex and both arms*	**∠A = ∠A (common)**
Angle sum of a triangle is 180°	*two angles known*	**∠C = 180° − ∠A − ∠B**
Linear pair adds to 180°	*angles on a straight line*	**∠ADE + ∠BDE = 180°**



X-shape
vertically opposite angles



Z-shape between ∥ lines
alternate angles



Shared vertex
common angle ∠A in both triangles

? WHY THIS STEP?

Why learn these as "clue → reason" pairs? Every similarity proof in this chapter needs two pairs of equal angles, and almost all of them come from this table. If you can see the X-shape, the Z-shape or the shared vertex, you already have half the proof.

✔ CHECK YOURSELF

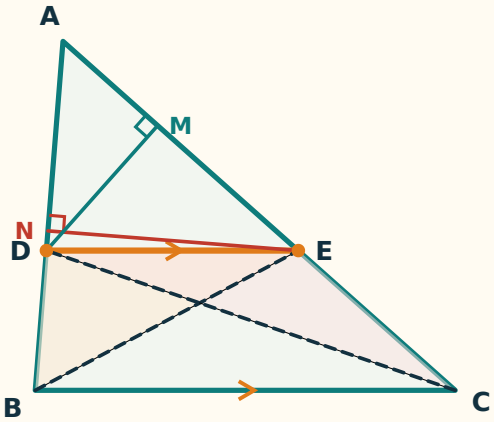
C2. In the figure above, suppose ∠1 = 62°. Find ∠2, ∠3 and ∠4, and name the fact you used for each.

Proving the Basic Proportionality Theorem

CORE

Learning objective: Write the prescribed proof of BPT, knowing the reason and the clue behind every line.

Hold this question: *How can areas of triangles prove something about lengths?*



GIVEN
In △ABC, DE ∥ BC; D on AB, E on AC.

TO PROVE
AD/DB = AE/EC

➤ CHOOSE THE METHOD

Construction. Join BE and CD. Draw EN ⊥ AB and DM ⊥ AC.

Idea. Triangles with the **same height** have areas in the ratio of their bases. Compare △ADE with △BDE (heights EN), and △ADE with △DEC (heights DM). The link: △BDE and △DEC have equal areas.

#	Statement	Reason	Clue that unlocked it
1	ar(ADE) = ½ × AD × EN; ar(BDE) = ½ × DB × EN	**area = ½ × base × height**	*EN ⊥ AB is a height for both*
2	ar(ADE)/ar(BDE) = AD/DB …(1)	**divide; ½ and EN cancel**	*same height EN*
3	ar(ADE) = ½ × AE × DM; ar(DEC) = ½ × EC × DM	**area = ½ × base × height**	*DM ⊥ AC is a height for both*
4	ar(ADE)/ar(DEC) = AE/EC …(2)	**divide; ½ and DM cancel**	*same height DM*
5	ar(BDE) = ar(DEC) …(3)	**triangles on the same base DE and between the same parallels DE and BC have equal areas**	*DE ∥ BC (given)*
6	ar(ADE)/ar(BDE) = ar(ADE)/ar(DEC)	**from (3): equal denominators**	—
7	Hence AD/DB = AE/EC. ∎	**from (1) and (2)**	*this is TO PROVE*

? WHY THIS STEP?

Where does "DE ∥ BC" get used? Only in line 5, and that is the whole point: two triangles on the same base DE with their third vertices B and C on a line parallel to DE have the same height, so equal areas. Without the parallel lines, line 5 fails and so does the theorem.

✓ EXAM TIP

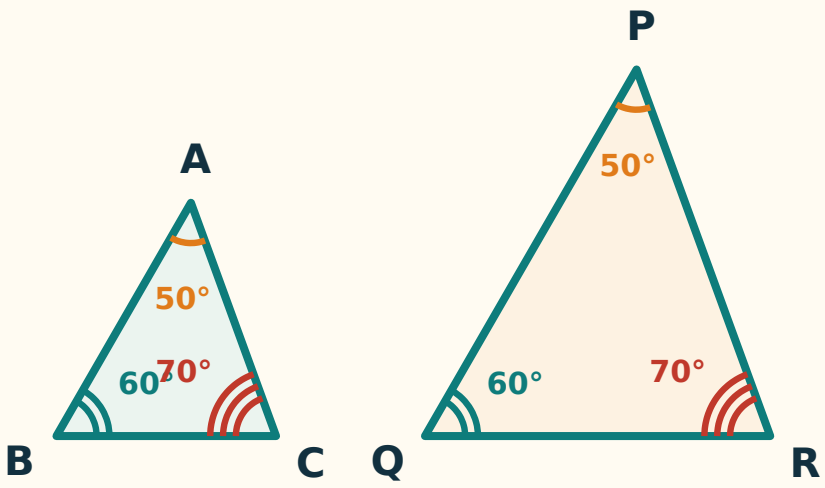
The syllabus asks you to **prove** this theorem, so learn the construction and the three area facts. Write the construction before the proof, number the results (1), (2), (3), and finish by stating the conclusion in the same words as TO PROVE.

AA similarity: two angles are enough

CORE

Learning objective: Use equal corresponding angles to prove two triangles similar, and know why two pairs suffice.

Hold this question: *Do I need all three pairs of angles, and is one pair ever enough?*



◆ UNDERSTAND

Theorem 6.3 (AAA criterion). If in two triangles the corresponding angles are equal, then their corresponding sides are in the same ratio, and the triangles are similar. **AA criterion:** if two angles of one triangle are equal to two angles of another, the triangles are similar, because the third angles must then be equal too (angle sum = 180°). (State and use; no proof required.)

◈ PROOF LINK

1 CLUE IN THE DIAGRAM	→	2 THEOREM THAT MATCHES	→	3 STEP IT LETS YOU WRITE
Two pairs of equal angles: from ∥ lines, an X-shape, a shared vertex, right angles, or given values		**AA similarity**: its condition is two pairs of equal corresponding angles		**△ABC ~ △PQR**, with vertices in matching order

Worked example: △ABC has ∠A = 50°, ∠B = 60°. △PQR has ∠P = 50°, ∠R = 70°. Are they similar?

#	Statement	Reason	Clue that unlocked it
1	∠C = 180° − 50° − 60° = 70°	**angle sum of a triangle**	*two angles of △ABC known*
2	∠Q = 180° − 50° − 70° = 60°	**angle sum of a triangle**	*two angles of △PQR known*
3	∠A = ∠P, ∠B = ∠Q (and ∠C = ∠R)	**values found**	*match equal angles*
4	△ABC ~ △PQR	**AA similarity criterion**	*two pairs of equal angles*

⚠ COMMON TRAP

One pair of equal angles is **not** enough. Two triangles can both contain a 50° angle and still have completely different shapes (Doubt 6). Find a **second** pair before writing "~".

✓ CHECK YOURSELF

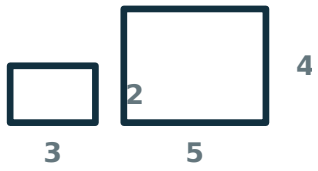
C8. In △ABC, ∠A = 35° and ∠B = 95°. In △PQR, ∠Q = 35° and ∠P = 50°. Show the triangles are similar and write the similarity statement in the correct vertex order.

Doubts 1 and 2: what similar means

DOUBT 1 *"These two rectangles look roughly the same shape. Doesn't that make them similar?"*

WHY IT HAPPENS	"Similar" in everyday English means "looks alike". In maths it is a precise condition.
SIMPLE EXPLANATION	Similar figures have equal corresponding angles **and** sides in one fixed ratio. You must check, not eyeball.
★ SPOT THE CLUE	Ask: is there **one** scale factor that turns every side into its partner?
WORKED EXAMPLE	Rectangles 3 × 2 and 5 × 4: 5/3 ≈ 1.67 but 4/2 = 2. No single scale factor, so **not similar**, even though all angles are 90°. *Why this step? Both conditions are needed for polygons.*
⚠ COMMON TRAP	Calling them similar because "both are rectangles". Equal angles alone are not enough for quadrilaterals.
✎ YOUR TURN DB1	Are a 4 × 6 rectangle and a 6 × 9 rectangle similar? Show the ratios.
✔ QUICK CHECK	Divide each side by its partner. All quotients equal → similar.

◉ VISUAL EXPLANATION

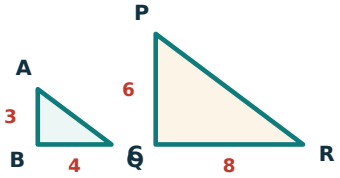


3 × 2 and 5 × 4: angles equal, ratios not.

DOUBT 2 *"If two triangles are similar, aren't they the same as congruent?"*

WHY IT HAPPENS	Both words describe "matching" triangles, and congruence was studied first (Class 9).
SIMPLE EXPLANATION	Congruent: same shape **and** same size (ratio 1). Similar: same shape, sizes in some ratio k. Congruent is the special case k = 1.
★ SPOT THE CLUE	Different lengths for matching sides → similar but not congruent.
WORKED EXAMPLE	Triangles with sides 3, 4, 5 and 6, 8, 10: ratios 6/3 = 8/4 = 10/5 = 2, so **similar, not congruent**. *Why this step? k ≠ 1.*
⚠ COMMON TRAP	Writing AB = PQ after proving △ABC ~ △PQR. Similarity gives AB/PQ = k, not equality.
✎ YOUR TURN DB2	True or false: (a) every pair of congruent triangles is similar; (b) every pair of similar triangles is congruent. Give a reason for each.
✔ QUICK CHECK	If the ratio is 1, they are congruent; otherwise only similar.

◉ VISUAL EXPLANATION



Sides 3, 4 → 6, 8: k = 2.

Decision tree: from question to theorem

Learning objective: Follow the branches to choose a theorem, and check its conditions before writing the step.

Mark the diagram. Then ask: what must I show or find?

↓

A length or ratio on the sides of a triangle with a ∥ line

↓

Is the line ∥ to a side and ending on the other two sides?

↓

Piece of a side → **BPT**

↓

The parallel side itself → **AA**, then DE/BC = AD/AB

↓

Not given ∥ → you cannot use BPT

Two lines are parallel

↓

Do you know lengths on two sides of a triangle?

↓

Equal ratios → **converse of BPT**

↓

Unequal ratios → **not parallel**

↓

Or: equal corresponding / alternate angles → parallel lines (Class 9 fact)

Triangles similar, or a ratio between two triangles

↓

What do you know about both triangles?

↓

Two pairs of angles → **AA**

↓

Three side ratios → **SSS** · angle + its two sides → **SAS**

↓

One angle only, or SSA → **not enough**

↓

Before you write the step: Are the theorem's conditions given or proved? Is the vertex order right? Does the ratio use corresponding sides? Does the last line say exactly what was asked?

? WHY THIS STEP?

Every branch starts from **what you must show**. That is the step most students skip, and it decides the theorem more reliably than any keyword. Questions often chain branches: prove similar (AA), then find a length (corresponding sides).

✎ YOUR TURN

Find the missing link. In △ABC, D is on AB and E on AC with AD/DB = AE/EC. To prove ∠ADE = ∠ABC, a student writes: (1) AD/DB = AE/EC (given); (2) _____ ; (3) ∠ADE = ∠ABC (corresponding angles). What must line (2) say, and which theorem justifies it? Hint: line (3) needs parallel lines. Answer in Section H, p. 46.

✓ CHECK YOURSELF

Try the tree: "In △PQR, S and T are on PQ and PR with PS/SQ = PT/TR. Prove ∠PST = ∠PQR." Which branches do you take, and in what order?

Mistake clinic (1 of 3)

MISTAKE 1 · Confusing similar with congruent	
✗ STUDENT'S WORK	*△ABC ~ △PQR and AB = 4 cm.* "So PQ = 4 cm."
⚠ WHERE IT FAILS	Similarity gives a **ratio**, not equal lengths. Only congruence (ratio 1) gives PQ = AB.
? WHY TEMPTING	Class 9 congruence proofs ended in "so AB = PQ", and the habit carries over.
✓ CORRECTED	PQ = k × AB, where k must be found from a known pair of corresponding sides.
➤ PREVENTION RULE	Similar ⇒ write a ratio. Congruent ⇒ write an equality.
✎ FIX IT E1	△ABC ~ △DEF, AB = 5, DE = 7.5 and BC = 6. A student writes EF = 6. Correct it.

MISTAKE 2 · Wrong vertex correspondence	
✗ STUDENT'S WORK	*∠A = 40°, ∠B = 70° in △ABC; ∠Q = 40°, ∠P = 70° in △PQR.* "△ABC ~ △PQR."
⚠ WHERE IT FAILS	That statement pairs A with P (40° with 70°). Equal angles say A ↔ Q and B ↔ P, so C ↔ R.
? WHY TEMPTING	Writing both triangles alphabetically looks tidy.
✓ CORRECTED	△**ABC** ~ △**QPR**. Then AB/QP = BC/PR = CA/RQ.
➤ PREVENTION RULE	Match the angle marks first, then write the letters in that order.
✎ FIX IT E2	∠X = 55°, ∠Y = 65° in △XYZ; ∠M = 65°, ∠N = 55° in △LMN. A student writes △XYZ ~ △LMN. Correct it.

MISTAKE 3 · Comparing non-corresponding sides	
✗ STUDENT'S WORK	*From △ABC ~ △PQR:* "AB/PQ = BC/PR."
⚠ WHERE IT FAILS	BC is positions 2–3, so its partner is QR, not PR.
? WHY TEMPTING	PR may look the right size or sit in the same place on the page.
✓ CORRECTED	AB/PQ = **BC/QR** = CA/RP.
➤ PREVENTION RULE	Partners use the same positions in the two names.
✎ FIX IT E3	From △DEF ~ △STU a student writes DE/ST = EF/SU. Find and correct the error.

Practice 1: reading diagrams and similar figures

Learning objective: Use notation, angle facts and the meaning of similarity correctly. (Level 1)

Level 1 | Level 2 | Level 3 | Level 4 | Level 5 *Answers: Section H.*

P1 In △PQR, name (a) the side opposite ∠Q, and (b) the angle included between sides QP and QR. In △ABC, name the angle included between AB and BC.

Purpose: read notation. easy ★☆☆

Working space (use separate paper if needed)

P2 True or false? (a) All congruent triangles are similar. (b) All similar triangles are congruent. (c) Any two equilateral triangles are similar. (d) Any two isosceles triangles are similar. (e) Any two squares are similar. (f) Any two rectangles are similar.

Purpose: use the definition and counter-examples. easy ★☆☆

Working space (use separate paper if needed)

P3 Give a counter-example: two quadrilaterals whose corresponding angles are equal but which are not similar. Draw them and state their sides.

Purpose: show that one condition is not enough. easy ★☆☆

Mark the diagram · Given · To prove

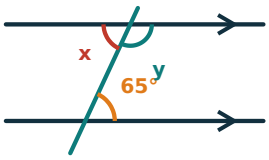
Working space (use separate paper if needed)

P4 The two horizontal lines are parallel. Find x and y, and give the reason for each.

Purpose: name angle facts as reasons. easy ★☆☆

Mark the diagram · Given · To prove

Working space (use separate paper if needed)



P5 A triangle has sides 3 cm, 4 cm and 6 cm. It is enlarged with scale factor 2.5. Find the sides of the new triangle. Are the two triangles congruent, similar, or both?

Purpose: use a scale factor. easy ★☆☆

Mark the diagram · Given · To prove

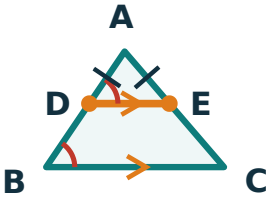
Working space (use separate paper if needed)

P6 From the marks on this diagram only, list (a) the pair of parallel lines, (b) the pair of equal angles, and (c) the pair of equal lengths. Which of these could you have *proved* if it were not marked?

Purpose: read marks as given information. moderate ★★☆

Mark the diagram · Given · To prove

Working space (use separate paper if needed)



Revision map

Learning objective: Recall every key fact, criterion and clue on one page.

◆ UNDERSTAND

Similar: same shape; corresponding angles equal **and** sides in one ratio k. **Congruent**: the case k = 1. All congruent figures are similar, not the other way round. For triangles, either condition alone is enough (that is what the criteria say).
△**ABC** ~ △**PQR** means A↔P, B↔Q, C↔R; AB/PQ = BC/QR = CA/RP.

BPT (prove)



DE ∥ BC ⇒ AD/DB = AE/EC; also AD/AB = AE/AC

Converse (state, use)



AD/DB = AE/EC ⇒ DE ∥ BC

AA



two pairs of equal angles

SSS



three side ratios equal (sort first)

SAS



equal angle + the two sides forming it, proportional

◈ PROOF LINK

1 CLUE IN THE DIAGRAM
∥ line in a triangle · X-shape · Z-shape · shared vertex · right angles · three sides · angle with its two sides

→

2 THEOREM THAT MATCHES
BPT · converse · AA · SSS · SAS (conditions checked)

→

3 STEP IT LETS YOU WRITE
a ratio, a ∥ line, or ~ with correct vertex order, each with its reason

When the question asks for…	Use
A piece of a side, with a ∥ line	BPT
The parallel side DE	AA, then DE/BC = AD/AB
"Show DE ∥ BC" with lengths	Converse of BPT
Prove ~ (angles)	AA
Prove ~ (sides)	SSS or SAS
A ratio like OA/OC = OB/OD	name the triangles from TO PROVE, then AA
Heights and shadows	AA, height ↔ height

⚠ COMMON TRAP

Final error checklist. ☐ Similar gives ratios, not equal sides (E1) ☐ Vertex order from the angles (E2) ☐ Partners by position (E3) ☐ Two pairs of angles, not one (E4) ☐ BPT vs converse named correctly (E5) ☐ Nothing assumed from appearance (E6) ☐ Criterion and its evidence written (E7) ☐ SAS angle included (E8) ☐ Ratios checked by substitution (E9)

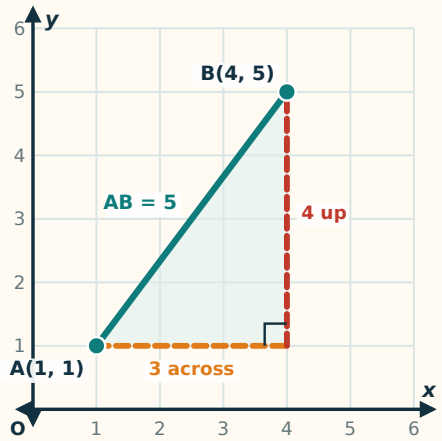
MATHS WITHOUT GUESSWORK

Coordinate Geometry

Understand the Points.
Choose the Formula.
Solve Independently.

A guided problem-solving workbook

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See the 3 and the 4 first. Then AB = 5.

SIGNATURE FEATURE

Understand the diagram before choosing the formula.

Every method in this book starts with a picture: where the points sit, what the run and rise are, which end of a segment a point is closer to. The formula comes last, so you know what it calculates and can check it against what you see.

Core topics: Distance formula · Section formula (internal division), with the mid-point as a special case

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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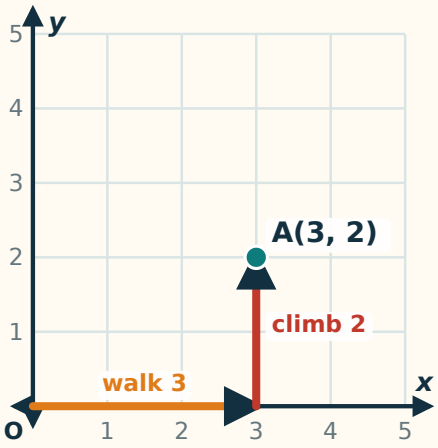
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Ordered pairs: why the order matters

PREREQUISITE

Learning objective: Plot (x, y) correctly every time, and see why (3, 2) and (2, 3) are different points.

Hold this question: *If both numbers are the same, does it matter which comes first?*



A(3, 2): walk 3, climb 2



B(2, 3): walk 2, climb 3

◆ UNDERSTAND

An **ordered pair** is two numbers in a fixed order. The first is always x (across), the second is always y (up/down). Swap them and you get a different point: A(3, 2) is lower and further right than B(2, 3).

Memory rule: walk, then climb. You walk along the corridor (x) before you take the lift (y). The alphabet helps too: x comes before y.

◉ DIAGRAM FIRST

1 SEE · (−1, 4): two moves to make.

2 READ · First number −1: walk 1 **left**. Second number 4: climb 4 **up**.

3 CHOOSE · Mark it. Then read it back from the grid to check: across −1, up 4 ✓.

★ SPOT THE CLUE

A point where both numbers are equal, like (3, 3), is the only kind where swapping does not matter. Every other swap moves the point.

? WHAT IF?

What if only the second number changes? (3, 2) → (3, 5): the walk stays 3, so the point moves straight **up** by 3. Changing only y moves a point vertically; changing only x moves it horizontally. You will use this idea for the shortcut on page 14.

⚠ COMMON TRAP

Reading a grid as "row first". In a classroom seat plan you might say "row 2, column 3". In coordinate geometry the across number (column) always comes first: (3, 2).

✓ CHECK YOURSELF

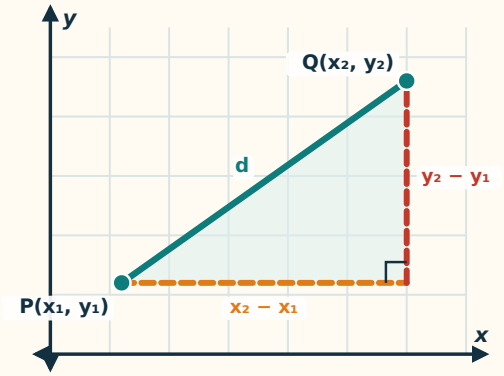
C2. (a) Write the point that is 2 units to the left of the y-axis and 5 units above the x-axis. (b) Are (4, 1) and (1, 4) the same point? Give a reason.

The distance formula

CORE

Learning objective: Write the distance formula, say what every symbol means, and use it in eight clear steps.

Hold this question: *What exactly does each part of the formula calculate in my sketch?*



◆ UNDERSTAND

Exactly the same triangle as on page 11, now with letters. The run is the change in x; the rise is the change in y. Pythagoras gives $d^2 = \text{run}^2 + \text{rise}^2$, so

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance from the origin O(0, 0) to P(x, y): $OP = \sqrt{x^2 + y^2}$

Symbol	Means	In the sketch
(x_1, y_1)	the point you call "first"	P
(x_2, y_2)	the point you call "second"	Q
$x_2 - x_1$	change in x (run)	horizontal leg
$y_2 - y_1$	change in y (rise)	vertical leg
d	straight-line distance PQ	slanted side

➤ CHOOSE THE METHOD

The eight-step routine. (1) Name the two points. (2) Label them: write $x_1 =$, $y_1 =$, $x_2 =$, $y_2 =$. (3) Find $x_2 - x_1$, with brackets round negatives. (4) Find $y_2 - y_1$. (5) Square both. (6) Add. (7) Take the square root. (8) Simplify and compare with your sketch.

? WHY THIS STEP?

Does it matter which point is "first"? No. Swapping the points changes the run from 3 to −3, but $(-3)^2 = 9 = 3^2$. Squaring removes the direction and keeps the size. So $(x_1 - x_2)^2 + (y_1 - y_2)^2$ gives the same answer.

Worked example: find the distance between (2, 3) and (5, 7).

#	Step	Why this step?
1	$x_1 = 2$, $y_1 = 3$, $x_2 = 5$, $y_2 = 7$	Label before substituting: it stops x and y getting mixed.
2	run $= 5 - 2 = 3$; rise $= 7 - 3 = 4$	Change in x, then change in y.
3	$d = \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$ units	Square, add, then take the root. 5 lies between 4 and 3 + 4 = 7 ✓.

✔ CHECK YOURSELF

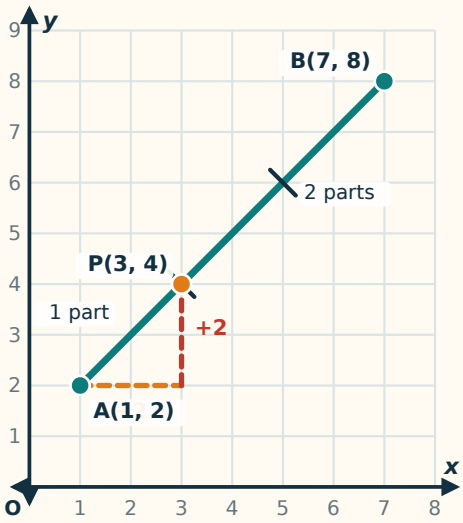
C8. Find the distance between (1, −1) and (7, 7). Sketch first and predict a range for the answer.

Section formula: see the fraction of the way

CORE

Learning objective: Understand what "divides in the ratio $m_1 : m_2$" means on a diagram, and find the point by walking part of the way.

Hold this question: *If P divides AB in the ratio 1 : 2, how far along from A is P?*



◉ DIAGRAM FIRST

1 SEE · P is on AB, closer to A. AP : PB = 1 : 2.

2 READ · Cut AB into 1 + 2 = 3 equal parts. P is after **1 part**: it is **1/3 of the way** from A to B.

3 CHOOSE · Walk 1/3 of the run and 1/3 of the rise from A.
Run = 7 − 1 = 6, so x = 1 + ⅓(6) = **3**.
Rise = 8 − 2 = 6, so y = 2 + ⅓(6) = **4**.
P = (3, 4).

◆ UNDERSTAND

Divides AB internally in the ratio $m_1 : m_2$ means P lies between A and B with AP : PB = $m_1 : m_2$. So P is $\frac{m_1}{m_1 + m_2}$ of the way from A to B. Read the ratio in the same order as the letters: the first number goes with the part next to A.

◆ UNDERSTAND

The mid-point is the 1 : 1 case. Halfway along means half the run and half the rise, which works out as the **average** of the coordinates:

$$\text{Mid-point of } A(x_1, y_1) \text{ and } B(x_2, y_2) = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$

Example: the mid-point of A(−3, 5) and B(5, −1) is ((−3 + 5)/2, (5 + (−1))/2) = **(1, 2)**. Check: from A, run 8 and rise −6; half of each is 4 and −3, and (−3 + 4, 5 − 3) = (1, 2) ✓.

⚠ COMMON TRAP

Thinking 1 : 2 means "P is halfway" or "P is ½ of the way". The ratio compares the two parts with each other, not a part with the whole. 1 : 2 → ⅓ of the way. 3 : 1 → ¾ of the way.

✓ CHECK YOURSELF

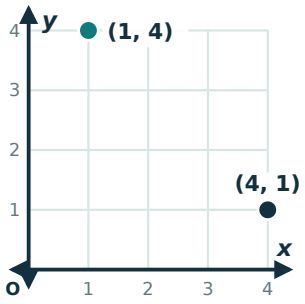
C11. Find the mid-point of (−4, 6) and (2, −2). Then check it by walking half the run and half the rise from (−4, 6).

Doubts 1 and 2: reading points

DOUBT 1 *"(3, 2) and (2, 3) use the same numbers. Aren't they the same point?"*

WHY IT HAPPENS	Both pairs contain a 3 and a 2, and everyday lists like {2, 3} do not care about order.
SIMPLE EXPLANATION	In (x, y) the **position** carries the meaning: first = across, second = up. Swapping the numbers swaps the moves.
★ SPOT THE CLUE	Brackets with a comma mean an **ordered** pair: read the first number as x every time.
WORKED EXAMPLE	Which is higher, (1, 4) or (4, 1)? Compare the **second** numbers: 4 > 1, so (1, 4) is higher. *Why this step? Height is the y-coordinate.*
⚠ COMMON TRAP	Plotting (1, 4) as "up 1, right 4". That is the point (4, 1).
✎ YOUR TURN DB1	Plot P(−1, 4) and Q(4, −1). Which one is in quadrant II?
✔ QUICK CHECK	Read your point back from the grid: across first, then up. It must match the pair you were given.

◉ VISUAL EXPLANATION

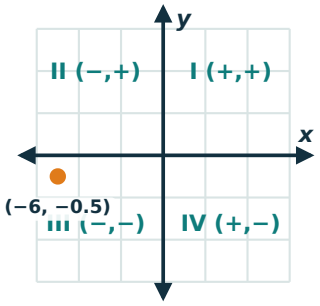


Same digits, different places.

DOUBT 2 *"How do I find the quadrant quickly without drawing anything?"*

WHY IT HAPPENS	Students try to memorise "quadrant II = ..." instead of picturing left/right and up/down.
SIMPLE EXPLANATION	Sign of x: + right, − left. Sign of y: + up, − down. The two signs name a corner of the plane.
★ SPOT THE CLUE	Ignore the sizes of the numbers. Only the two **signs** matter.
WORKED EXAMPLE	(−6, −0.5): x negative → left; y negative → down. Left and down → **quadrant III**. *Why this step? A tiny −0.5 is still below the axis.*
⚠ COMMON TRAP	Calling (5, −2) quadrant II because "one sign is negative, so it's the 2nd". It is right and down: IV.
✎ YOUR TURN DB2	Without plotting, name the quadrants of (−7, −0.5) and (0.2, −9).
✔ QUICK CHECK	Draw a quick "+" sign in the margin and point to the corner your two signs describe.

◉ VISUAL EXPLANATION



Anticlockwise from top right.

Decision tree: from question to formula

Learning objective: Follow the branches to choose a method, and know which check to run before you finish.

◉ **Sketch the points first.** Then ask: what is the answer going to be?

A number of units (a length)

Do the points share x or y?

Yes → subtract: bigger − smaller

No → distance formula $\sqrt{(\text{run}^2 + \text{rise}^2)}$

Shape / collinear / equidistant? → find **all** needed distances and compare or equate

A point (x, y)

Is a ratio given?

1 : 1 or "mid-point", "centre", "bisect" → mid-point formula

$m_1 : m_2$, trisection, "AP = ⅖ AB" → section formula

No ratio, but "equidistant" → unknown point + distance equation

A ratio or an unknown value

What is known about the point?

The point is given → k : 1, use x, check with y

"The x-axis divides" → y = 0; "the y-axis" → x = 0

"Distance is d" → square both sides; expect two answers

Before you finish, always check: Does the answer agree with my sketch? Distance between the larger change and the sum of changes? Dividing point between A and B? Every unknown substituted back? Did I answer the exact question (a point, a ratio, or a length with units)?

⚠ COMMON TRAP

A decision tree is a guide, not a machine. A question can mix ideas: "find the ratio in which the y-axis divides AB, and the length of AB" needs a ratio **and** a distance. Work through each part separately.

? WHY THIS STEP?

Every branch starts from the **type of answer** (length, point or ratio). That is the question students skip most often, and it decides the formula more reliably than any keyword.

✓ CHECK YOURSELF

Try the tree: which branch would you take for "Find the centre of a circle whose diameter has ends (−3, 2) and (5, 6)"? (Hint: what kind of answer is a centre?)

Mistake clinic (1 of 3)

MISTAKE 1 · Reversing the ordered pair

✗ STUDENT'S WORK	*Plot B(2, 5).* The student marks the point 5 to the right and 2 up.
⚠ WHERE IT FAILS	The student plotted (5, 2). The first number is always the move across.
? WHY TEMPTING	Both numbers are positive, so the point still "looks" reasonable in quadrant I.
✓ CORRECTED	B(2, 5): walk 2 right, then climb 5 up. B is higher than it is far across.
➤ PREVENTION RULE	Walk, then climb. Read the point back from the grid before moving on.
✎ FIX IT E1	A student plots (5, −1) by going 1 right and 5 down. Write the correct moves. Is the quadrant affected?

MISTAKE 2 · Wrong direction for a negative, or the wrong region

✗ STUDENT'S WORK	*Plot (−4, 3), and name the region of (0, −2).* The student plots (4, 3), and says "(0, −2) is in quadrant IV".
⚠ WHERE IT FAILS	The minus on −4 was ignored, so the point landed on the wrong side of the y-axis. And (0, −2) has x = 0, so it is **on** the y-axis, in no quadrant.
? WHY TEMPTING	Negative signs are small and easy to miss; and (0, −2) is "below and slightly right-looking" on a rough sketch.
✓ CORRECTED	(−4, 3) is 4 **left**, 3 up: quadrant II. (0, −2) lies on the negative y-axis.
➤ PREVENTION RULE	Say the sign aloud as a direction ("minus four: left four"). A zero coordinate means: on an axis.
✎ FIX IT E2	A student writes "(−4, 0) lies in quadrant II." Correct the statement and explain.

MISTAKE 3 · Substituting coordinates in the wrong places

✗ STUDENT'S WORK	*Distance between (−1, 4) and (3, 1).* $\sqrt{((3-4)^2 + (1-(-1))^2)} = \sqrt{(1+4)} = \sqrt{5}$.
⚠ WHERE IT FAILS	$y_1 = 4$ was put in the x-bracket and $x_1 = -1$ in the y-bracket. The run and rise were mixed up.
? WHY TEMPTING	All four numbers appear somewhere, so the working looks complete.
✓ CORRECTED	run = 3 − (−1) = 4, rise = 1 − 4 = −3. d = $\sqrt{(16+9)} = \sqrt{25} = \mathbf{5}$.
➤ PREVENTION RULE	Fill an x_1, y_1, x_2, y_2 table first (Doubt 16). Only x's in the first bracket.
✎ FIX IT E3	Find the error: the distance between (2, 7) and (5, 3) is written as $\sqrt{((5-7)^2 + (3-2)^2)}$. Correct it.

Practice 1: points and the plane

Learning objective: Read, write and place points confidently, including points on the axes. (Level 1)

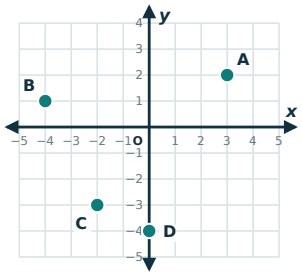
Level 1 | Level 2 | Level 3 | Level 4 | Level 5

Answers: Section H.

P1 Write the coordinates of A, B, C and D in the grid. For each, name its quadrant, or the axis it lies on.

Purpose: read points from a diagram. easy ★☆☆

Working space (use separate paper if needed)



P2 Without plotting, say where each point lies (quadrant or axis): (−6, 3), (4, −4), (0, 7), (−2, −9), (5, 0), (8, 1).

Purpose: use the signs, not the sizes. easy ★☆☆

Sketch first

Working space (use separate paper if needed)

P3 Write the coordinates of the point that is: (a) 3 units left of the y-axis and 4 units below the x-axis; (b) on the x-axis, 6 units to the left of the origin; (c) on the y-axis, 2.5 units above the origin.

Purpose: turn words into ordered pairs. easy ★☆☆

Sketch first

Working space (use separate paper if needed)

P4 True or false? Give a reason for each. (a) (2, −5) and (−5, 2) are the same point. (b) Every point on the y-axis has x-coordinate 0. (c) The origin lies in quadrant I.

Purpose: spot the common misconceptions. easy ★☆☆

Working space (use separate paper if needed)

P5 P is (4, −3). Q has the same y-coordinate as P, but its x-coordinate has the opposite sign. Write Q, name the quadrants of P and Q, and find the length PQ.

Purpose: see how a sign change moves a point. moderate ★★☆

Sketch first

Working space (use separate paper if needed)

P6 Find the length of each segment without the distance formula: (a) (−5, 4) to (7, 4); (b) (−1, −8) to (−1, −2). Say which is horizontal and which is vertical.

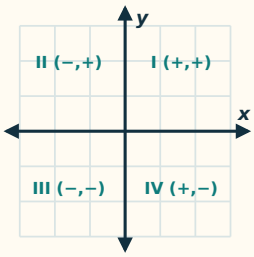
Purpose: use the shortcut correctly. easy ★☆☆

Sketch first

Working space (use separate paper if needed)

Revision map

Learning objective: Recall every key fact, formula and check on one page.

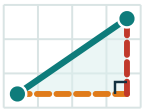


◆ UNDERSTAND

(x, y): walk x (± = right/left), then climb y (± = up/down). x = distance from the y-axis; y = distance from the x-axis.
On the x-axis: (x, 0). **On the y-axis**: (0, y). **Origin**: (0, 0). Points on an axis are in no quadrant.
Same y → horizontal, length = bigger x − smaller x. **Same x** → vertical, length = bigger y − smaller y.

Distance

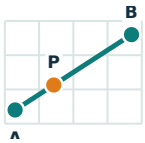
$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$



run² + rise², then root. From O: √(x² + y²).

Section (internal)

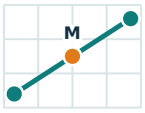
$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2} , \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)$$



AP : PB = $m_1 : m_2$. Criss-cross. Unknown ratio → k : 1.

Mid-point (1 : 1)

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$



Average. Diagonals of a parallelogram share it; a centre is it.

◉ DIAGRAM FIRST

1 SEE · Sketch the points. Quadrants? Which is higher, which further right?

2 READ · Is the answer a length, a point, a ratio or a value? Run/rise? Fraction of the way? Shared x or y?

3 CHOOSE · Shortcut, distance, section (k : 1), or mid-point. Then substitute with brackets.

When the question asks for…	Use
A length between two points	Shortcut if x or y is shared; else distance formula
Triangle type, square, collinear	All distances, then compare
Equidistant / unknown coordinate	Unknown as (x, 0), (0, y) or (x, y); $PA^2 = PB^2$; keep ± answers
Point in a given ratio / trisection	Section formula; check the fraction of the way
"In what ratio" / an axis divides	k : 1; set the known coordinate; check the other
Mid-point, centre, parallelogram	Average, forwards or backwards

⚠ COMMON TRAP

Final error checklist. ☐ x before y (E1) ☐ Negative = left/down; zero = on an axis (E2) ☐ x's with x's (E3) ☐ Brackets round negatives (E4) ☐ $(-a)^2$ is positive (E5) ☐ Square, add, then root (E6) ☐ A difference is a distance only for horizontal/vertical (E7) ☐ Criss-cross the ratio (E8) ☐ Both answers, final line answers the question (E9)

✓ CHECK YOURSELF

Answer-checking routine: (1) Does the answer agree with the sketch? (2) Distance: more than the larger change, less than their sum. (3) Dividing point: between A and B, at the right fraction of the way. (4) Unknowns: substitute back. (5) Mid-point: equal distances to both ends.

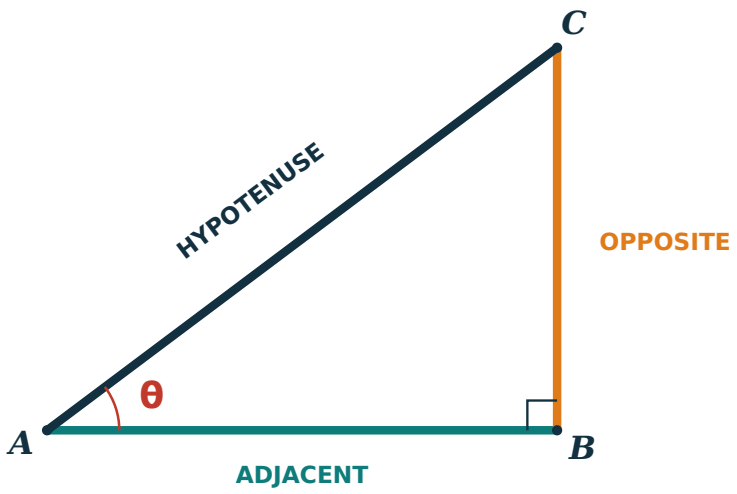
MATHS WITHOUT GUESSWORK

Introduction to Trigonometry

Understand the Triangle.
Choose the Ratio.
Solve Independently.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27



For angle θ: sin θ = 3/5 · cos θ = 4/5 · tan θ = 3/4

INSIDE THIS BOOK

Name the sides from the angle
Choose sin, cos or tan
Derive the standard values
Use $\sin^2 A + \cos^2 A = 1$
Prove simple identities

> Signature feature:
> **Identify the sides before choosing the ratio**

Core topics: trigonometric ratios of acute angles · values at 0°, 30°, 45°, 60°, 90° · $\sin^2 A + \cos^2 A = 1$ and simple identities

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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Opposite and adjacent depend on the angle

CORE

Learning objective: Name the opposite and adjacent sides relative to a chosen acute angle, and see why they swap.

Once the hypotenuse is fixed, the other two sides get their names **from the angle you are looking at**. Call that angle the **reference angle** (often θ, read “theta”).

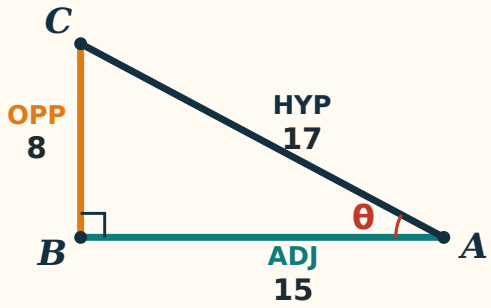
Opposite side

The side that **faces** the reference angle. It does **not touch** the angle’s corner.

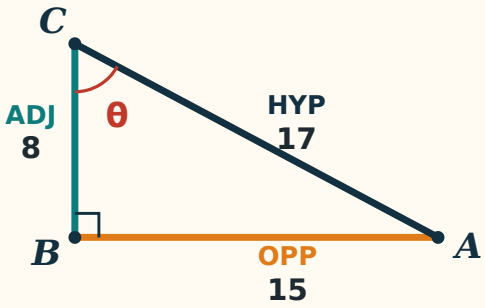
Adjacent side

The side that **touches** the reference angle and is **not** the hypotenuse. (“Adjacent” means “next to”.)

Same triangle, different angle



Reference angle A



Reference angle C

	From angle A	From angle C	Changes?
Hypotenuse	AC = 17	AC = 17	never: it is fixed by the right angle
Opposite	BC = 8	AB = 15	yes: swaps
Adjacent	AB = 15	BC = 8	yes: swaps

▲ IDENTIFY THE SIDES BEFORE CHOOSING THE RATIO — the three-question routine

Role	Side	How I know
Hypotenuse	**Q1**	Where is the right angle? The side across from it is the hypotenuse.
Opposite	**Q2**	Which side does the reference angle look across at (without touching)?
Adjacent	**Q3**	Which remaining side touches the reference angle?

Ask the three questions **in this order**. Naming the hypotenuse first stops you calling it “adjacent”, because the hypotenuse also touches the reference angle.

⚠ COMMON TRAP

Calling the hypotenuse “adjacent” because it touches the angle. Both AC and AB touch angle A, but AC is already the hypotenuse, so the adjacent side is AB.

✔ CHECK YOURSELF

C2. In ΔPQR, right-angled at R, name the hypotenuse, and the opposite and adjacent sides (a) for angle P, (b) for angle Q. What do you notice?

Concept 2 · sine, cosine and tangent

CORE

Learning objective: Define sin, cos and tan as pairs of named sides, and use them on a labelled triangle.

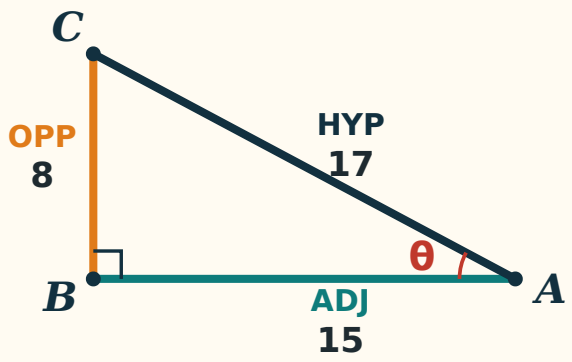
For an acute angle θ in a right triangle, three ratios compare the three pairs of sides. Each ratio is named by the **pair of sides** it uses, measured from θ.

sine	cosine	tangent
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

What they mean. sin θ: how much of the slope is "rise". cos θ: how much of the slope is "run". tan θ: rise per unit of run, the steepness. Because the hypotenuse is the longest side, **sin θ and cos θ are less than 1** for an acute angle; tan θ can be any positive number.

Memory aid (after you understand it): SOH sine = opposite/hypotenuse · **CAH** cosine = adjacent/hypotenuse · **TOA** tangent = opposite/adjacent. The mnemonic only works if the sides were named correctly first.

Worked example 1 · all three sides known



ΔABC, right angle at B.

Question. In ΔABC, right-angled at B, AB = 15 cm, BC = 8 cm and AC = 17 cm. Find sin A, cos A and tan A.

What do I notice first? The right angle is at B, so AC is the hypotenuse.

Solution. sin A = BC/AC = **8/17**; cos A = AB/AC = **15/17**; tan A = BC/AB = **8/15**.

Check: $8^2 + 15^2 = 64 + 225 = 289 = 17^2$ ✓; sin and cos are below 1 ✓.

▲ IDENTIFY THE SIDES BEFORE CHOOSING THE RATIO — angle A in ΔABC

Role	Side	How I know
Hypotenuse	**AC = 17**	opposite the right angle at B
Opposite	**BC = 8**	faces angle A; does not touch A
Adjacent	**AB = 15**	touches A; it is not the hypotenuse

Why these ratios? sin uses OPP & HYP → 8/17 · cos uses ADJ & HYP → 15/17 · tan uses OPP & ADJ → 8/15.

✓ CHECK YOURSELF

Explain it in your own words. Why is the hypotenuse never the *top* of a sine or cosine fraction? (Think about the answer being bigger than 1.)

✓ EXAM TIP

In your answer, write the ratio in words before numbers, e.g. "sin A = BC/AC = 8/17". The letters show the examiner you named the sides.

✎ YOUR TURN

T1 (transfer). In the same triangle, find sin C, cos C and tan C. Name the sides from angle C first. What do you notice when you compare with sin A and cos A?

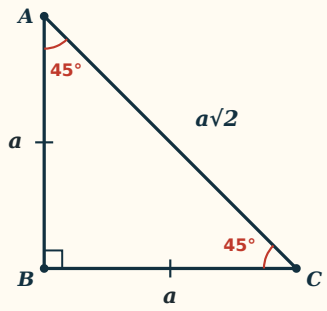
Concept 5 · Standard values from two triangles

CORE

Learning objective: Derive the ratios of 30°, 45° and 60° from the half-square and half-equilateral triangles.

The values for 30°, 45° and 60° are not things to memorise blindly. Each comes from one simple triangle you can redraw in ten seconds.

45°: half a square



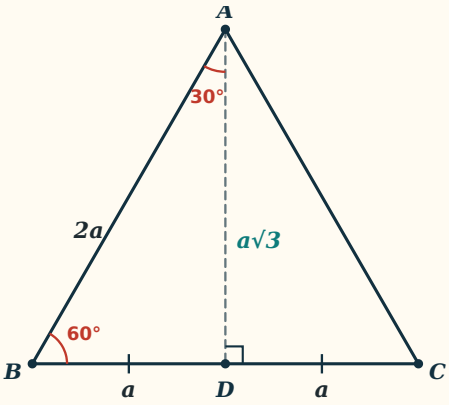
Isosceles right triangle.

In ΔABC right-angled at B with ∠A = ∠C = 45°, the two legs are equal (equal angles face equal sides): AB = BC = a. Pythagoras: AC = √(a² + a²) = a√2.

From 45° (either angle): opposite = a, adjacent = a, hypotenuse = a√2.

$$\sin 45° = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \cos 45° = \frac{1}{\sqrt{2}} \cdot \tan 45° = \frac{a}{a} = 1$$

30° and 60°: half an equilateral triangle



Equilateral ABC, AD ⊥ BC.

Take equilateral ΔABC with side 2a; every angle is 60°. Drop AD ⊥ BC. Triangles ABD and ACD are congruent (RHS), so BD = a and ∠BAD = 30°. In right ΔABD: AD = √((2a)² − a²) = a√3.

From **30°** (at A): opp BD = a, adj AD = a√3, hyp AB = 2a. From **60°** (at B): opp AD = a√3, adj BD = a, hyp 2a.

$$\sin 30° = \frac{1}{2} \cdot \cos 30° = \frac{\sqrt{3}}{2} \cdot \tan 30° = \frac{1}{\sqrt{3}} \quad \sin 60° = \frac{\sqrt{3}}{2} \cdot \cos 60° = \frac{1}{2} \cdot \tan 60° = \sqrt{3}$$

▲ IDENTIFY THE SIDES BEFORE CHOOSING THE RATIO — ΔABD, the 30°–60°–90° triangle

Role	Side	How I know
Hypotenuse	**AB = 2a**	opposite the right angle at D, for both acute angles
Opposite	**30°: BD = a · 60°: AD = a√3**	the side each angle faces
Adjacent	**30°: AD = a√3 · 60°: BD = a**	the side each angle touches (not AB)

The same two legs swap roles between 30° and 60°. That is why sin 30° = cos 60° = 1/2 and cos 30° = sin 60° = √3/2.

⚠ COMMON TRAP

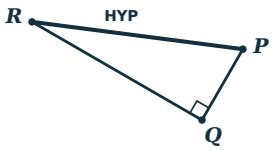
Thinking the shortest side is opposite 60°. The shortest side (a) is opposite the **smallest** angle, 30°. Bigger angle, bigger opposite side.

✓ CHECK YOURSELF

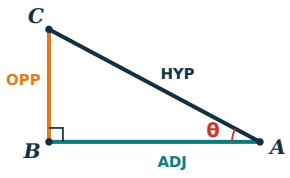
C10. Without the table, derive cosec 60°, sec 30° and cot 45° from the triangles above. Rationalise where needed.

Doubts 1–2: hypotenuse, opposite, adjacent

DOUBT 1 *"In a tilted triangle I can't tell which side is the hypotenuse."*

✗ WHY THIS CONFUSION HAPPENS	Students learn the hypotenuse as "the slanting side", which only works when the triangle sits on a flat base.
◆ UNDERSTAND	The hypotenuse is the side **opposite the right angle**, and it is always the longest side. Find the small square first; the side not touching that corner is the hypotenuse.
★ SPOT THE CLUE	The right-angle mark, or the words "right-angled at Q".
➤ CHOOSE THE RATIO / IDENTITY	No ratio yet. Naming the hypotenuse comes before every ratio.
✦ SOLVED EXAMPLE	ΔPQR is right-angled at Q, drawn tilted. The side not touching Q is PR, so PR is the hypotenuse, even though it is drawn vertically and looks "straight". 
? WHY THIS STEP?	If the hypotenuse is wrong, every ratio built on it is wrong.
⚠ COMMON TRAP	Picking the longest-looking or most slanted line. Figures are not drawn to scale.
✎ YOUR TURN DB1	ΔKLM is right-angled at L. Name the hypotenuse. If KL = 9 and LM = 12, find its length.
✔ CHECK YOURSELF	Your answer must be longer than both 9 and 12.

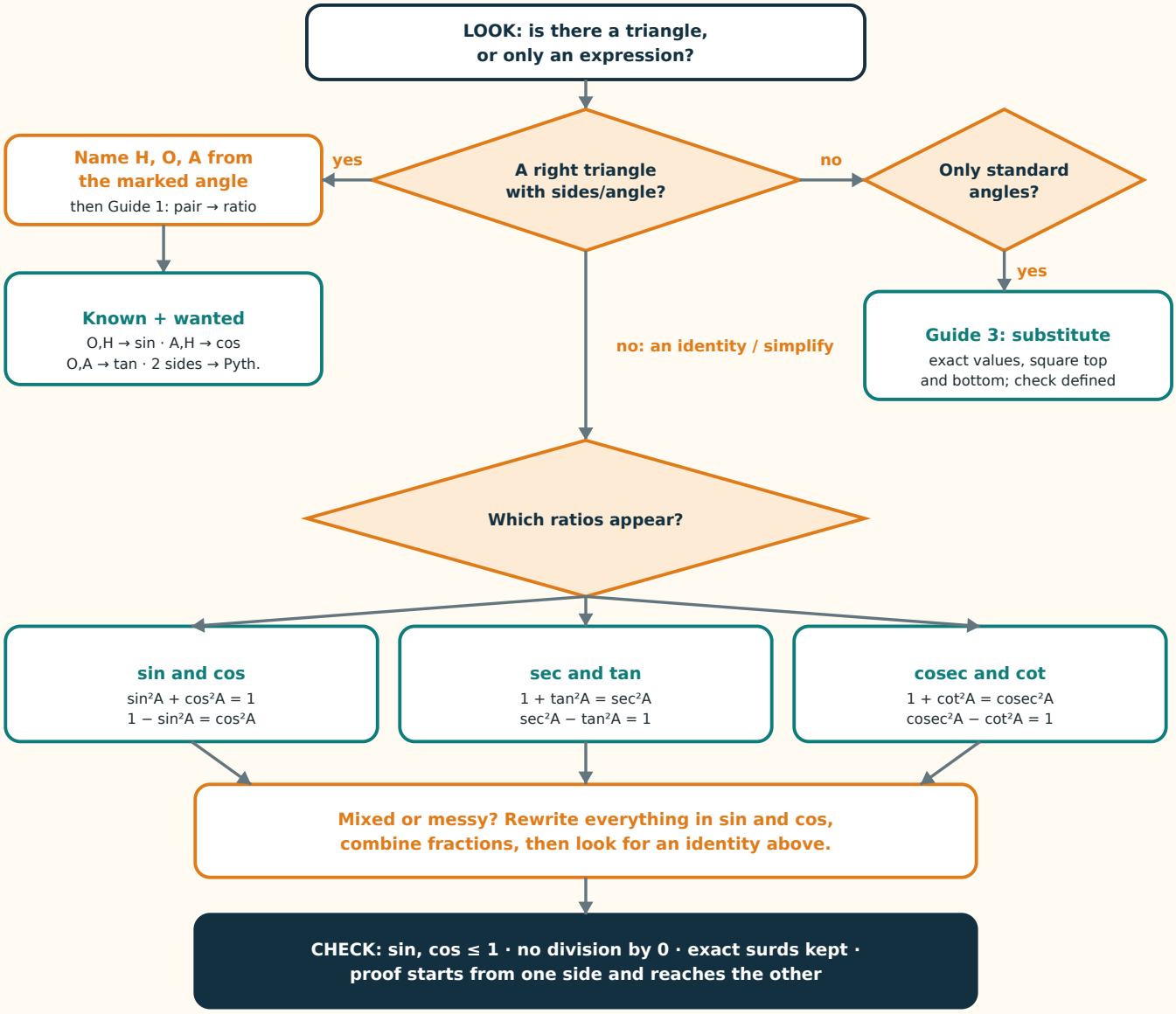
DOUBT 2 *"How do I tell the opposite side from the adjacent side?"*

✗ WHY THIS CONFUSION HAPPENS	Both shorter sides "touch" the triangle; the words feel vague.
◆ UNDERSTAND	From the reference angle: the **opposite** side is the one it faces and does **not touch**; the **adjacent** side touches it. The hypotenuse also touches it, so name the hypotenuse first and exclude it.
★ SPOT THE CLUE	The marked angle (θ or a letter). Put your finger on that corner.
➤ CHOOSE THE RATIO / IDENTITY	Once O and A are named, the ratio follows from the pair needed.
✦ SOLVED EXAMPLE	Right angle at B, angle A marked, AB = 15, BC = 8, AC = 17. A touches AB and AC; AC is the hypotenuse, so **adjacent = AB**. A does not touch BC, so **opposite = BC**. 
? WHY THIS STEP?	Excluding the hypotenuse first leaves exactly one touching side: the adjacent.
⚠ COMMON TRAP	Calling AC adjacent because it touches A.
✎ YOUR TURN DB2	In ΔXYZ right-angled at Z with angle X marked, name the opposite and adjacent sides.
✔ CHECK YOURSELF	The opposite side never contains the letter of the reference angle.

Ratio and identity flowchart

Learning objective: Follow a decision path from the question to the first correct step.

Start at the top. Each diamond is a question about the problem in front of you; each box is an action you already know. A question can pass through two branches, so look again after the first step.



Trace two questions through the chart

Question	Path
Right ΔABC, ∠B = 90°, ∠A = 30°, BC = 4. Find AC.	triangle → name sides: BC opp, AC hyp → O,H → sin: AC = 4 ÷ ½ = 8
Evaluate $\tan^2 60° - \sec^2 60°$.	no triangle → standard angles → substitute: 3 − 4 = −1 (also the identity 1 + tan² = sec² gives −1 ✓)

✓ CHECK YOURSELF

Try the chart. “Simplify $(1 + \tan^2 A)\cos^2 A$.” Which boxes do you pass through? (Answer: no triangle → not standard angles → sec and tan → $1 + \tan^2 A = \sec^2 A$, then $\sec^2 A \cos^2 A = 1$.)

Mistake clinic (1 of 3)

MISTAKE 1 · Wrong hypotenuse

✗ STUDENT'S WORK	*"Right angle at B, AB = 3, BC = 4, AC = 5. sin A = AC/BC = 5/4."*
⚠ WHERE IT FAILS	AC is the hypotenuse and belongs on the bottom of a sine. A sine above 1 is impossible.
? WHY TEMPTING	AC is the longest number, and the student wrote the biggest number first.
✓ CORRECTED	sin A = opp/hyp = BC/AC = 4/5.
➤ PREVENTION RULE	**Find the right angle; the side across from it is the hypotenuse.**
✎ FIX IT E1	A student writes cos C = 13/12 in a 5-12-13 triangle. Explain the error and correct it.

MISTAKE 2 · Sides not renamed for a new angle

✗ STUDENT'S WORK	*"From A: opp = BC, adj = AB. So tan C = BC/AB too."*
⚠ WHERE IT FAILS	From C, the sides swap: opposite is AB and adjacent is BC.
? WHY TEMPTING	The same triangle is still on the page, so the old labels feel valid.
✓ CORRECTED	tan C = AB/BC.
➤ PREVENTION RULE	**New angle → new O and A. Only H stays.**
✎ FIX IT E2	In ΔPQR (right angle at Q), a student uses sin R = QR/PR. Is that right? Which ratio of R is QR/PR?

MISTAKE 3 · Ratio chosen from a pattern

✗ STUDENT'S WORK	*"∠A = 30°, hypotenuse 10, want the adjacent side. tan 30° = adj/10."*
⚠ WHERE IT FAILS	tan uses opposite and adjacent; the hypotenuse is not in tan at all.
? WHY TEMPTING	The previous example used tan, so the student repeated it.
✓ CORRECTED	Adjacent and hypotenuse → cos: cos 30° = adj/10, adj = 10 × √3/2 = 5√3.
➤ PREVENTION RULE	**Name the two sides involved, then pick the ratio with exactly that pair.**
✎ FIX IT E3	∠P = 60°, opposite side 9 cm, want hypotenuse. A student uses cos. Choose correctly and solve.

MISTAKE 4 · Mixing up sin and cos values

✗ STUDENT'S WORK	*"sin 60° = 1/2."*
⚠ WHERE IT FAILS	In the 30-60-90 triangle, 60° faces the longer leg √3, so sin 60° = √3/2. The value 1/2 is sin 30° (= cos 60°).
? WHY TEMPTING	The two values 1/2 and √3/2 are swapped in memory.
✓ CORRECTED	sin 60° = √3/2; cos 60° = 1/2.
➤ PREVENTION RULE	**Sketch the 1-√3-2 triangle: the shortest side faces the smallest angle.**
✎ FIX IT E4	Write sin 30°, cos 30°, sin 60°, cos 60° from the triangle, not from memory.

Practice 1 · Recognise

Learning objective: Identify sides, match ratios, recognise standard values and identities.

1 Recognise | 2 Apply | 3 Connect | 4 Board-style

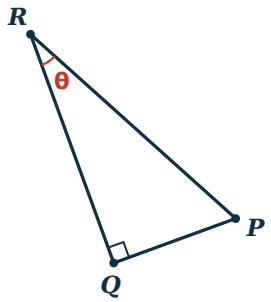
Answers: Section H

P1 In ΔABC, right-angled at C, the hypotenuse is
(A) AB (B) BC (C) AC (D) it depends on the marked angle

Purpose: Find the hypotenuse from the right angle. **easy** ★☆☆

Working space (use separate paper if needed)

P2 In the figure, the right angle is at Q and angle R is marked. Name the hypotenuse, the side opposite R and the side adjacent to R. Then name the side opposite P.



Purpose: Name sides relative to an angle. **easy** ★☆☆

Working space (use separate paper if needed)

P3 Which ratio equals hypotenuse ÷ adjacent?
(A) sin θ (B) cosec θ (C) sec θ (D) cot θ

Purpose: Match a ratio to its sides. **easy** ★☆☆

Working space (use separate paper if needed)

P4 Which of these equals ½?
(A) sin 60° (B) cos 60° (C) tan 45° (D) sin 45°

Purpose: Recognise a standard value. **easy** ★☆☆

Working space (use separate paper if needed)

P5 True or false? Give a reason. (a) tan A is always less than 1. (b) sec A = 12/5 for some acute angle A. (c) cos A is short for the cosecant of A. (d) sin θ = 4/3 for some angle θ.

Purpose: Test the size of each ratio. **moderate** ★★☆

Working space (use separate paper if needed)

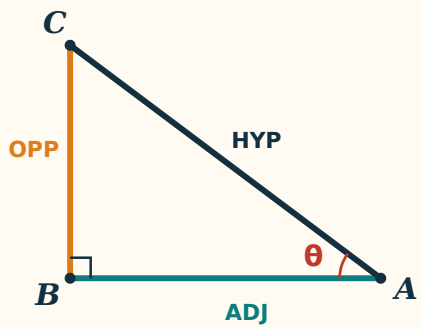
P6 Match each expression with a single ratio, naming the identity: (a) $1 - \sin^2 A$ (b) $\sec^2 A - 1$ (c) $\operatorname{cosec}^2 A - \cot^2 A$.

Purpose: Recognise identities. **easy** ★☆☆

Working space (use separate paper if needed)

One-page revision sheet

Learning objective: Recall every ratio, the standard values, the identities and the side-naming habit on one page.



Ratio	Sides from θ	Ratio	Sides from θ
sin	O/H	cosec = 1/sin	H/O
cos	A/H	sec = 1/cos	H/A
tan	O/A	cot = 1/tan	A/O

tan = sin/cos · cot = cos/sin · sin, cos ≤ 1 · sec, cosec ≥ 1

∠A	0°	30°	45°	60°	90°
sin	0	1/2	1/√2	√3/2	1
cos	1	√3/2	1/√2	1/2	0
tan	0	1/√3	1	√3	n.d.
cosec	n.d.	2	√2	2/√3	1
sec	1	2/√3	√2	2	n.d.
cot	n.d.	√3	1	1/√3	0

n.d. = not defined (division by 0). Rebuild 30°/60° from the 1-√3-2 triangle and 45° from 1-1-√2.

◆ UNDERSTAND

Identities (from Pythagoras):

$\sin^2 A + \cos^2 A = 1$

$1 + \tan^2 A = \sec^2 A \quad (0° \le A < 90°)$

$1 + \cot^2 A = \operatorname{cosec}^2 A \quad (0° < A \le 90°)$

Enrichment only: $\sin(90° - A) = \cos A$, $\cos(90° - A) = \sin A$, $\tan(90° - A) = \cot A$.

▲ IDENTIFY THE SIDES BEFORE CHOOSING THE RATIO — the habit to keep

Role	Side	How I know
Hypotenuse	1st	the side across from the right angle (longest)
Opposite	2nd	the side the marked angle faces (does not touch)
Adjacent	3rd	the side the marked angle touches, not the hypotenuse

Then choose: O + H → sin · A + H → cos · O + A → tan · unknown on the bottom → use the reciprocal · two sides, no angle → Pythagoras.

★ SPOT THE CLUE

Evaluating: identify → substitute in brackets → square top and bottom → common denominators → check for 1/0. **Proving:** start from the harder side, rewrite in sin and cos, combine fractions, apply an identity, cancel only factors, end with "= RHS".

⚠ COMMON TRAP

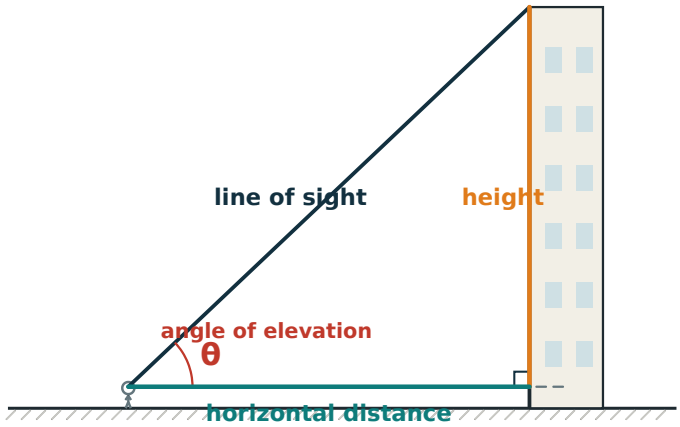
Final error checklist. ☐ wrong hypotenuse (E1) ☐ sides not renamed (E2) ☐ ratio by habit (E3) ☐ sin/cos values swapped (E4) ☐ ratio vs reciprocal (E5) ☐ $\sin^2 A$ misread (E6) ☐ half-squared fraction (E7) ☐ root/fraction slip (E8) ☐ value for "not defined" (E9) ☐ sin(A + B) split (E10) ☐ cancelled a term (E11) ☐ assumed the result (E12)

MATHS WITHOUT GUESSWORK

Some Applications of Trigonometry

Understand the Situation.
Draw the Diagram.
Solve Independently.

A guided problem-solving workbook · Heights and Distances
CBSE Class 10 Mathematics · Syllabus session 2026–27



tan θ = height ÷ horizontal distance

INSIDE THIS BOOK

Read the story
Draw the right triangle
Place the angle correctly
Choose sin, cos or tan
Solve and check

> Signature feature:
> **Word problem → correct diagram → correct ratio**

Core topics: angle of elevation · angle of depression · heights and distances with at most two right triangles · 30°, 45°, 60°

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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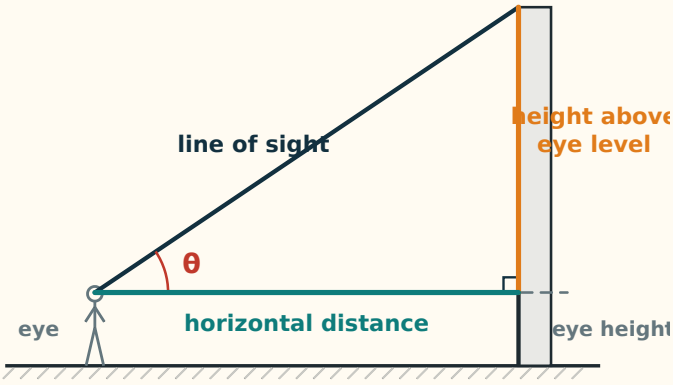
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Concept 2 · Height, horizontal distance, line of sight

CORE

Learning objective: Distinguish the three lengths in every problem and know their roles in the right triangle.

Every problem in this chapter talks about some of these quantities. Naming them correctly is the first half of the work.



θ = angle of elevation, between the dashed horizontal through the eye and the line of sight. Not to scale.

Quantity	What it is	In the right triangle
Height	vertical distance from the base (or from eye level) up to the top of the object	usually **opposite** the angle at the observer
Horizontal distance	distance along level ground (or along the eye-level horizontal) between observer and object	usually **adjacent** to the angle at the observer
Line of sight	straight line from the observer's eye to the point being looked at	always the **hypotenuse**
Eye height	height of the observer's eye above the ground	added to the height above eye level to get the full height

◆ UNDERSTAND

The three lengths form one right triangle. The horizontal and the vertical meet at 90°; the line of sight joins their ends. So line of sight² = height² + horizontal distance², and the line of sight is always the longest of the three.

⚠ COMMON TRAP

Using the line of sight as the ground distance. "The observer is 40 m from the tower" means the **horizontal** distance to the foot of the tower, unless the question says "40 m from the top" or gives a ladder/string length.

✓ CHECK YOURSELF

C2. A girl whose eyes are 1.5 m above the ground looks at the top of a 21.5 m pole. What is the height of the pole *above her eye level*? Which side of her right triangle is this?

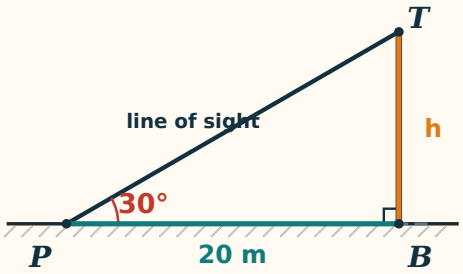
Method 1 · Height from a distance and an angle

CORE

Learning objective: Find a vertical height when the horizontal distance and the angle of elevation are known.

When to use: the horizontal distance to the object and the angle of elevation are known; the height is wanted.

Question. From a point P on level ground, 20 m from the foot B of a vertical pole BT, the angle of elevation of the top T is 30°. Find the height of the pole.



Not to scale.

▤ READ THE SITUATION

What is happening in the story? A person at P looks **up** at the top of a pole 20 m away. The ground is level and the pole is vertical, so the pole meets the ground at 90°.

What do I know?

- PB = 20 m (horizontal)
- ∠TPB = 30° (elevation)
- ∠TBP = 90° (vertical pole on level ground)

What must I find?

BT = h (vertical height)

▲ WORD PROBLEM → CORRECT DIAGRAM → CORRECT RATIO Method 1

Role	The words say…	…so in the diagram
OBJECT	*"a vertical pole BT"*	vertical line BT; right angle at B
ADJACENT	*"20 m from the foot B"*	horizontal PB = 20 m
ANGLE	*"the angle of elevation of the top T is 30°"*	∠TPB = 30°, at P, between PB (horizontal) and PT (sight)
UNKNOWN	*"find the height of the pole"*	BT = h, opposite the 30° angle

Correct ratio: BT is **opposite** 30°, PB is **adjacent**; PT is not given or wanted → **tan 30° = BT/PB**.

Solve. $\tan 30° = h/20 \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20} \Rightarrow h = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$ m $\approx \frac{20 \times 1.732}{3} \approx$ **11.55 m**.

✓ CHECK YOURSELF

Check whether the answer makes sense. At 30° (less than 45°) the height must be **less** than the horizontal distance: 11.55 m < 20 m ✓. At 45° they would be equal; above 45° the height is bigger.

? WHY THIS STEP?

Why tan and not sin? sin 30° = opposite/hypotenuse needs PT, the line of sight, which is neither given nor asked. tan uses exactly the two sides in the problem: the known PB and the wanted BT.

✎ YOUR TURN

T1. From a point on the ground 40 m from the foot of a flagpole, the angle of elevation of its top is 30°. Draw the diagram, then find the height of the flagpole (√3 ≈ 1.732).

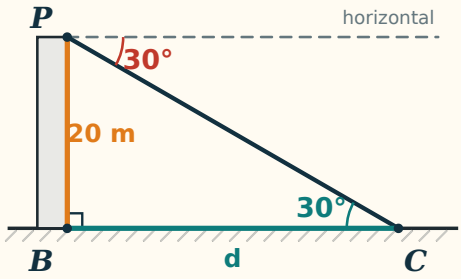
Method 4 · Problems with an angle of depression

CORE

Learning objective: Place the angle of depression correctly and move it inside the triangle with alternate angles.

When to use: the observer is high up (top of a building, cliff, tower, balloon) looking **down**. The depression angle is outside the triangle; alternate angles bring it inside.

Question. From the top P of a 20 m high building, the angle of depression of a scooter C on the road is 30°. Find the distance of the scooter from the foot B of the building.



Horizontal at P ∥ road BC.

READ THE SITUATION

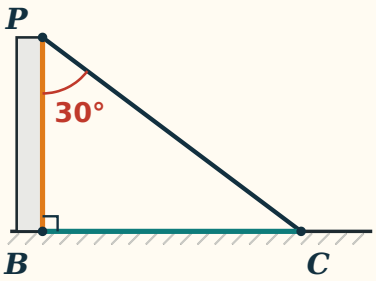
What is happening? Someone on the roof looks **down** at a scooter. Their eye line starts horizontal (parallel to the road) and tilts down 30°.

WORD PROBLEM → CORRECT DIAGRAM → CORRECT RATIO — Method 4

Role	The words say…	…so in the diagram
OBSERVER	*"from the top P of a 20 m high building"*	vertical PB = 20 m; draw a horizontal through P
ANGLE	*"the angle of depression of a scooter is 30°"*	between the horizontal at P and the sight line PC
PARALLEL	*"(horizontal at P ∥ road)"*	alternate angles: ∠PCB = 30° inside ΔPBC
UNKNOWN	*"distance of the scooter from the foot"*	BC = d, adjacent to the 30° at C

Correct ratio: From C: PB **opposite**, BC **adjacent** → **tan 30° = PB/BC**: $1/\sqrt{3} = 20/d$, $d = 20\sqrt{3} \approx$ **34.64 m**.

Spot the wrong diagram



A student's diagram.

COMMON TRAP

What is wrong? The student marked 30° at P **between the building and the line of sight**. That is the angle from the vertical, which equals 90° − 30° = 60°. With this wrong angle the student writes tan 30° = BC/20 and gets BC = $20/\sqrt{3} \approx 11.55$ m, which is far too short. The depression angle is measured from the **horizontal**.

SPOT THE CLUE

Words that signal depression: "from the top of", "looks down at", "angle of depression of", "observed from a balloon / aeroplane / cliff". The observer is **above** the object.

YOUR TURN

T6. From the top of a 45 m cliff, the angle of depression of a boat is 60°. Draw the diagram (mark both the depression angle and the alternate angle), then find the distance of the boat from the foot of the cliff.

Doubts 1 and 2

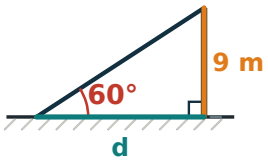
DOUBT 1 — *"There's no diagram in the question. How do I even start drawing?"*

✗ WHY THIS CONFUSION HAPPENS
Students try to picture the whole scene at once, with buildings, people and sky, instead of the few lines that matter.

◆ UNDERSTAND
You only need three lines: the **vertical object**, the **horizontal ground (or eye level)**, and the **line of sight**. They always make one right triangle.

★ SPOT THE CLUE
Nouns tell you the objects (tower, pole, tree = vertical); "from a point on the ground" tells you where the observer is; "angle of elevation/depression" tells you where the angle goes.

✎ DRAW AND LABEL
Draw the ground, stand the object up on it, put the observer on the ground, draw the sight line to the top, mark 90° at the foot and the angle at the observer.



➤ CHOOSE THE RATIO
Pole (opposite) and ground distance (adjacent) → **tan**.

✦ SOLVED EXAMPLE
"A 9 m pole; from a point on the ground the angle of elevation of its top is 60°. How far is the point from the pole?" $\tan 60° = 9/d \Rightarrow d = 9/\sqrt{3} = 3\sqrt{3} \approx$ **5.20 m**.

? WHY THIS STEP?
Drawing first shows that 9 m is opposite 60° and d is adjacent, before any formula is written.

⚠ COMMON TRAP
Writing $\tan 60° = d/9$ because 9 was mentioned first. Position in the sentence means nothing; position in the diagram decides.

✎ YOUR TURN DB1
From a point 25 m from the foot of a tree, the angle of elevation of its top is 45°. Draw, then find the height.

✓ CHECK YOURSELF
At 45° the height and distance are equal.

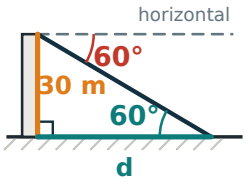
DOUBT 2 — *"What exactly is the difference between elevation and depression?"*

✗ WHY THIS CONFUSION HAPPENS
Both are "the angle with the line of sight", so students mix them up.

◆ UNDERSTAND
Both are measured from the **horizontal through the observer's eye**. **Elevation**: the object is above, you look up. **Depression**: the object is below, you look down.

★ SPOT THE CLUE
Where is the observer compared with the object? Below → elevation. Above (on a roof, cliff, tower) → depression.

✎ DRAW AND LABEL
For depression, draw the horizontal at the top, mark the angle under it, then copy it to the ground point (alternate angles).



➤ CHOOSE THE RATIO
Height (opposite) and ground distance (adjacent) → **tan**.

✦ SOLVED EXAMPLE
From the top of a 30 m tower, the angle of depression of a car is 60°. $\tan 60° = 30/d \Rightarrow d = 30/\sqrt{3} = 10\sqrt{3} \approx$ **17.32 m**.

? WHY THIS STEP?
The depression angle at the top equals the elevation angle at the car, so the same triangle works.

⚠ COMMON TRAP
Putting 60° at the top *inside* the triangle (between the tower and the sight line). That angle is 30°.

✎ YOUR TURN DB2
A bird on top of a 15 m pole sees a worm on the ground at an angle of depression of 30°. How far is the worm from the pole?

✓ CHECK YOURSELF
Draw the horizontal at the bird first.

Guide 4 · From words to equations

Learning objective: Follow one word problem through five stages: words, known, unknown, equation, answer.

Here is one word problem carried all the way to an answer. Each stage uses only what the previous stage produced.

1 · WORDS
"A man 1.5 m tall stands 25√3 m from a tower. The angle of elevation of the top of the tower from his eyes is 30°. Find the height of the tower."

2 · KNOWN
eye height 1.5 m · horizontal DE = 25√3 m · ∠ADE = 30°

3 · UNKNOWN
AE (above eye level), then AB = AE + 1.5

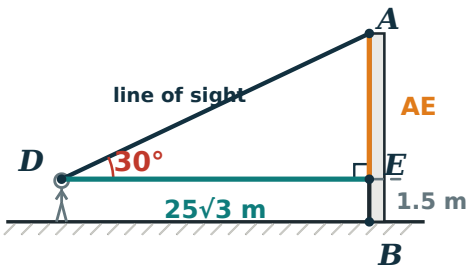
4 · EQUATION
tan 30° = AE/DE ⇒ 1/√3 = AE/(25√3)

5 · ANSWER
AE = 25√3/√3 = 25 m ⇒ AB = 25 + 1.5 = **26.5 m**

▲ WORD PROBLEM → CORRECT DIAGRAM → CORRECT RATIO

words → diagram

Role	The words say…	…so in the diagram
OBSERVER	*"a man 1.5 m tall … from his eyes"*	eye D, 1.5 m up; horizontal DE
ADJACENT	*"stands 25√3 m from a tower"*	DE = 25√3 m
ANGLE	*"angle of elevation … is 30°"*	∠ADE = 30° at the eye
UNKNOWN	*"height of the tower"*	AB = AE + 1.5



Correct ratio: AE **opposite**, DE **adjacent** → **tan 30°**.

◇ FIND THE MISSING LINK

A student wrote stages 1, 2, 4 and 5 but skipped stage 3. Their answer was "25 m". What did skipping the "unknown" stage cost them? (They forgot that the triangle's height is only the part above eye level, so 1.5 m was never added.)

A second run-through: looking down

1 · WORDS
"From the top of a 60 m building, the angle of depression of a car is 60°. How far is the car?"

2 · KNOWN
height 60 m; depression 60° ⇒ angle at car 60° (alternate)

3 · UNKNOWN
horizontal distance d (adjacent to 60° at the car)

4 · EQUATION
tan 60° = 60/d

5 · ANSWER
d = 60/√3 = 20√3 ≈ **34.64 m**

✓ EXAM TIP

In the exam, write stages 2 and 3 as two short lines: "Given: …" and "To find: …". It takes seconds, shows the examiner your reading of the question, and stops you from answering the wrong unknown.

✓ CHECK YOURSELF

Try it. Carry this problem through all five stages on rough paper: "From a point 20 m from the foot of a tower, the angle of elevation of its top is 60°. Find the length of the line of sight to the top." (Answer: 40 m, using cos 60° = 20/l.)

Mistake clinic (1 of 3)

MISTAKE 1 · Angle measured from the vertical

✗ STUDENT'S WORK	*"The line of sight makes 60° with the tower, so the angle of elevation is 60°."*
⚠ WHERE IT FAILS	Angles of elevation and depression are measured from the **horizontal**. The angle with the vertical tower is the complement.
? WHY TEMPTING	The tower is drawn clearly and the horizontal is not.
✓ CORRECTED	Angle of elevation = 90° − 60° = 30°.
➤ PREVENTION RULE	**Measure from the horizontal through the eye.**
✎ FIX IT E1	The line of sight to a pole's top makes 60° with the vertical pole. The observer is 15 m from the pole. Find the height.

MISTAKE 2 · Elevation drawn at the wrong vertex

✗ STUDENT'S WORK	*"30° elevation of a 20 m tower's top: tan 30° = d/20." (30° marked at the top of the tower)*
⚠ WHERE IT FAILS	The angle of elevation is at the **observer**, not at the top of the object. At the top the angle is 60°.
? WHY TEMPTING	The angle is "of the top", so it feels like it belongs at the top.
✓ CORRECTED	At the observer: tan 30° = 20/d ⇒ d = 20√3 ≈ 34.64 m.
➤ PREVENTION RULE	**The vertex of the angle is always the observer's eye.**
✎ FIX IT E2	Redo it correctly. What does the student's 20/√3 actually represent?

MISTAKE 3 · Depression put inside the triangle at the top

✗ STUDENT'S WORK	*"From a 35 m building, depression 60°: tan 60° = d/35."*
⚠ WHERE IT FAILS	60° is between the horizontal and the sight line, **outside** the triangle. Inside, at the top, the angle is 30°.
? WHY TEMPTING	The top is where the observer stands, so the student puts the angle there, inside.
✓ CORRECTED	Alternate angle 60° at the object: tan 60° = 35/d ⇒ d = 35/√3 ≈ 20.21 m.
➤ PREVENTION RULE	**Depression → draw the horizontal at the top → move the angle to the object.**
✎ FIX IT E3	From the top of a 45 m building, the angle of depression of a car is 30°. A student writes d = 45 tan 30°. Correct it.

MISTAKE 4 · Height confused with line of sight

✗ STUDENT'S WORK	*"Kite string 50 m at 30° to the ground, so the height is 50 tan 30°."*
⚠ WHERE IT FAILS	50 m is the **string** (hypotenuse), not the ground distance. tan needs opposite and adjacent.
? WHY TEMPTING	"Heights use tan" is applied without checking the sides.
✓ CORRECTED	Height = 50 sin 30° = 25 m.
➤ PREVENTION RULE	**A slanting length is the hypotenuse: use sin or cos.**
✎ FIX IT E4	A 24 m wire from the top of a pole makes 60° with the ground. A student writes h = 24 tan 60°. Correct it.

Practice 1 · Recognise

Learning objective: Identify angles, lengths and ratios, and match words with diagrams.

1 Recognise | 2 Apply | 3 Connect | 4 Board-style

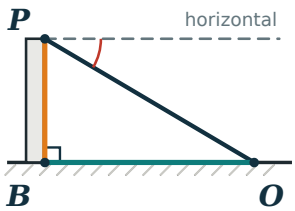
Answers: Section H

P1 The angle of elevation is the angle between
(A) the object and the line of sight (B) the horizontal and the line of sight
(C) the ground and the object (D) the line of sight and the object's top

Purpose: Recall the definition. **easy ★☆☆**

Working space (use separate paper if needed)

P2 In the figure, P is the top of a building and O is a car. Name (a) the angle of depression, (b) the height, (c) the horizontal distance, (d) the line of sight, (e) the angle inside ΔPBO equal to the angle of depression, with a reason.



Purpose: Read a diagram. **easy ★☆☆**

Working space (use separate paper if needed)

P3 In a kite problem, the string is the
(A) height (B) horizontal distance (C) hypotenuse (D) angle of elevation

Purpose: Identify the slant. **easy ★☆☆**

Working space (use separate paper if needed)

P4 Without calculating, name the ratio: (a) height of a tower and distance from it; (b) kite string and kite height; (c) ladder length and distance of its foot from the wall.

Purpose: Choose the ratio. **easy ★☆☆**

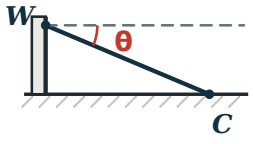
Working space (use separate paper if needed)

P5 From the top of a cliff, the angle of depression of a boat is 30°. The angle of elevation of the top of the cliff from the boat is
(A) 60° (B) 30° (C) 90° (D) 45°

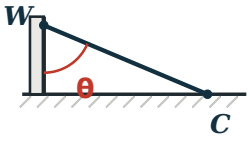
Purpose: Alternate angles. **easy ★☆☆**

Working space (use separate paper if needed)

P6 Which diagram shows "from a window W 8 m above the ground, the angle of depression of a car C is θ" correctly? Explain.



(i)



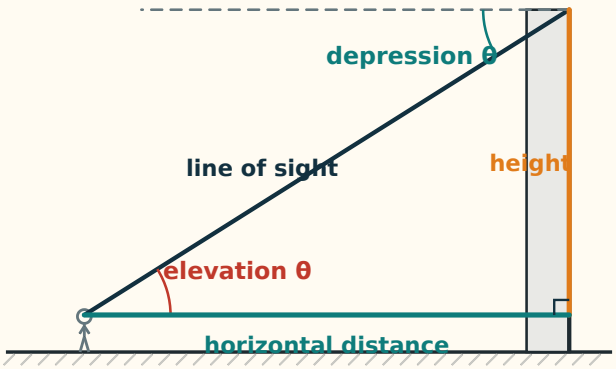
(ii)

Purpose: Spot the wrong diagram. **easy ★☆☆**

Working space (use separate paper if needed)

One-page revision sheet

Learning objective: Recall the definitions, the reference diagram, the ratios, the routine and the assumptions on one page.



Both angles are measured from a horizontal line. For the same line of sight they are equal (alternate angles). Not to scale.

Term	Meaning
Line of sight	eye → point viewed; the hypotenuse
Angle of elevation	horizontal → line of sight, object above the eye
Angle of depression	horizontal → line of sight, object below the eye
Height	vertical; usually opposite
Horizontal distance	along level ground; usually adjacent

Sides	Ratio	30°	45°	60°
O & A	**tan**	1/√3	1	√3
O & H	**sin**	1/2	1/√2	√3/2
A & H	**cos**	√3/2	1/√2	1/2

◆ UNDERSTAND

Routine: READ → VISUALISE → DRAW → LABEL → CHOOSE → SOLVE → CHECK.
Assumptions: level ground; vertical objects; straight lines of sight; observer height ignored unless given; one vertical plane.

★ SPOT THE CLUE

Two triangles? Find the shared side: height (two positions) or horizontal distance (object on top; up and down from a roof). Same side → subtract distances; opposite sides → add.

⚠ COMMON TRAP

Final error checklist. ☐ angle from the vertical (E1) ☐ angle at the wrong vertex (E2) ☐ depression inside the triangle (E3) ☐ slant used as height (E4) ☐ ground as hypotenuse (E5) ☐ ratio before labelling (E6) ☐ unjustified equal angles (E7) ☐ eye height forgotten (E8) ☐ same/opposite side ignored (E9) ☐ rearranging (E10) ☐ early rounding (E11) ☐ no unit / impossible answer (E12)

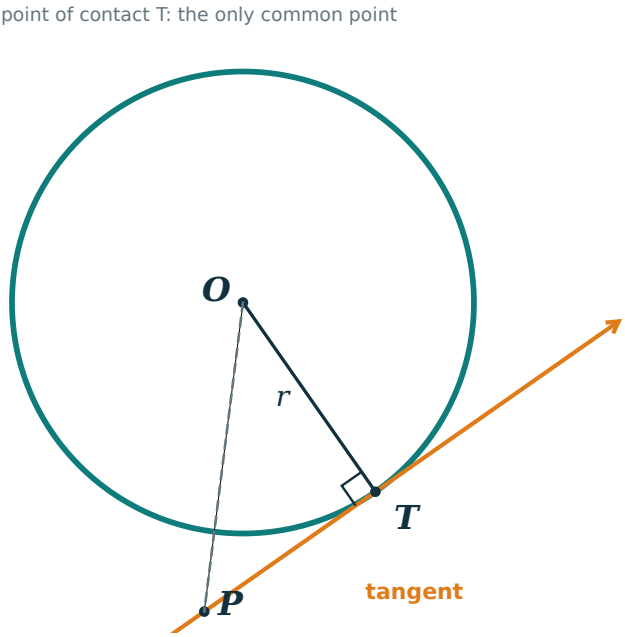
MATHS WITHOUT GUESSWORK

Circles

Understand the Diagram.
Recognise the Theorem.
Solve Independently.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27



OT ⊥ PT: the radius meets the tangent at 90°.

INSIDE THIS BOOK

Read a circle diagram
Spot the clue
Check a theorem's conditions
Write a proof with reasons
Solve new questions alone

Signature feature:
Why does this theorem apply here?

Core topics: tangent to a circle at the point of contact · tangent ⊥ radius · equal tangents from an external point

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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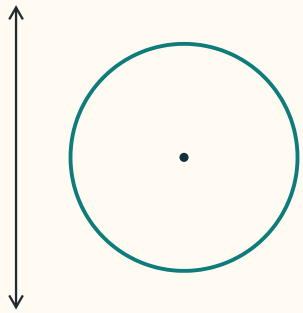
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A line and a circle: three possibilities

CORE

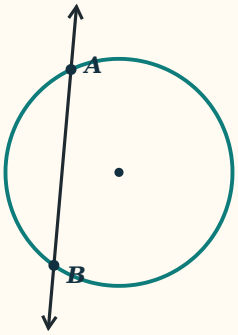
Learning objective: Classify a line as non-intersecting, secant or tangent, and define the point of contact.

Hold this question: *How many points can a straight line and a circle share, and why does "exactly one" matter so much?*



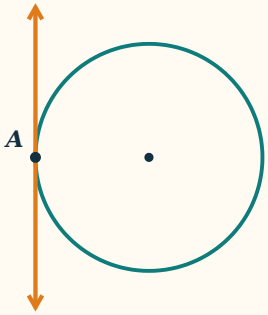
Non-intersecting line

(i) no common point



Secant

(ii) two common points

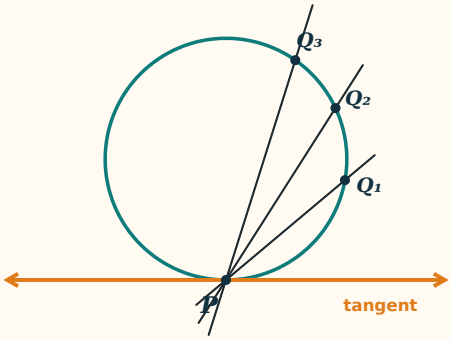


Tangent

(iii) exactly one common point

A line and a circle in the same plane share **0, 1 or 2 points**, never 3: a straight line can enter a circle once and leave once. The three cases have names:

- **Non-intersecting line**: no common point.
- **Secant**: exactly two common points (it cuts the circle; the part inside is a chord).
- **Tangent**: exactly one common point. That point is the **point of contact**, and the tangent is said to **touch** the circle there.



Turn a secant about P: Q slides towards P.

◆ UNDERSTAND

A tangent is the last position of a secant. Fix P on the circle and turn the line PQ about P. The second point Q slides along the circle towards P. When Q reaches P, the two common points have become one: the line is now the tangent at P. Turning a little further gives a secant again, so **at each point of a circle there is exactly one tangent**.

★ SPOT THE CLUE

The words **"touches"**, **"tangent at T"**, **"point of contact"**, and **"circle inscribed in / polygon circumscribing a circle"** (every side touches the circle) all tell you a line is a tangent. "Cuts", "meets at A and B", or "intersects" with two points named tell you it is a secant.

⚠ COMMON TRAP

Deciding from the picture. A secant that cuts very close to the edge can *look* like a tangent. Use only what the question states (or what you prove). Diagrams in exams are not drawn to scale.

✓ CHECK YOURSELF

C2. Classify each line and give a reason: (a) line *a* meets the circle at X and Y; (b) line *b* touches the circle at Z; (c) line *c* passes through the centre O. Then say why no line can meet a circle in three points.

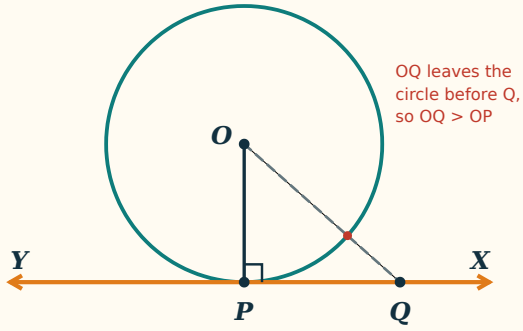
Theorem 1 · Tangent ⊥ radius at the point of contact

CORE

Learning objective: Explain why Theorem 1 is true, prove it, and state exactly when it applies.

The situation. On a rolling bicycle wheel, the spoke that points to the ground always stands straight up from the road. The spoke is a radius; the road is a tangent; they meet at 90°.

Why it makes sense. Stand at the centre O and look at the tangent line. Every point of the tangent except P is outside the circle, so it is *farther* from O than r. P is the only point at distance exactly r. So P is the **nearest** point of the line to O, and the shortest path from a point to a line is the perpendicular one.



XY touches the circle at P; Q is any other point of XY.

> **Theorem 1.** The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given: a circle with centre O and a tangent XY touching it at P. **To prove:** OP ⊥ XY.

#	Statement	Reason
1	Take any point Q on XY other than P, and join OQ.	Construction, to compare OQ with OP.
2	Q lies outside the circle.	If Q were inside or on the circle, XY would meet the circle at a second point and be a secant, not a tangent.
3	So OQ > r, that is, OQ > OP.	A point outside is farther from O than the radius; OP = r because P is on the circle.
4	This is true for **every** point Q of XY except P.	Q was any point other than P.
5	So OP is the shortest distance from O to the line XY.	From rows 3 and 4.
6	Hence OP ⊥ XY. ∎	Of all segments from a point to a line, the perpendicular is the shortest.

★ **WHY DOES THIS THEOREM APPLY HERE?** Theorem 1

Condition the theorem needs	Here?	How I know (evidence)
The line is a **tangent** (exactly one common point)	✓ **yes**	stated in the question, or proved
The radius goes to the **point of contact** of that tangent	✓ **yes**	the radius must end where the tangent touches

Then and only then: the radius and the tangent meet at 90°. A radius to any other point of the circle gets **no** right angle from this theorem.

◆ **UNDERSTAND**

In practice the theorem hands you a right angle at every point of contact. A right angle means a right triangle, and a right triangle means Pythagoras, RHS congruence and angle facts become available. Remark: since the perpendicular to OP at P is unique, there is exactly one tangent at each point of a circle.

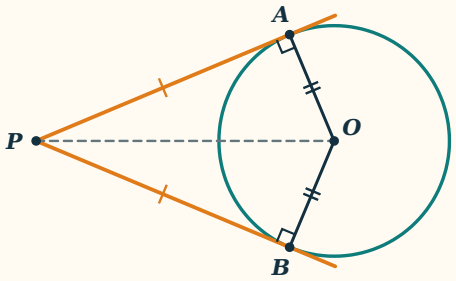
Theorem 2 · Tangents from an external point are equal

CORE

Learning objective: Prove that PA = PB, naming the correspondence and the congruence rule.

The situation. From a point P outside a circle, draw both tangents. Measure PA and PB: they always match. Why? The whole figure is **symmetrical about the line OP**. Fold the page along OP and the circle lands on itself, so the tangent PA lands exactly on PB.

A fold is a good idea, not a proof. The proof turns the symmetry into two congruent right triangles.



PA, PB touch the circle at A, B. OA, OB, OP joined.

Theorem 2. The lengths of tangents drawn from an external point to a circle are equal.

Given: a circle with centre O; P is outside; PA and PB are tangents touching the circle at A and B. **To prove:** PA = PB. **Construction:** join OA, OB and OP.

#	Statement	Reason
1	∠OAP = 90°	OA is the radius through the point of contact of tangent PA (Theorem 1)
2	∠OBP = 90°	OB is the radius through the point of contact of tangent PB (Theorem 1)
3	In right triangles OAP and OBP: OA = OB	radii of the same circle
4	OP = OP, and OP is the hypotenuse of both	common side; it is opposite the right angle in each
5	ΔOAP ≅ ΔOBP, with A ↔ B, O ↔ O, P ↔ P	RHS: right angle, hypotenuse, one side (rows 1–4)
6	PA = PB ∎	CPCT: PA and PB are corresponding sides

★ **WHY DOES THIS THEOREM APPLY HERE?** RHS congruence inside Theorem 2

Condition the theorem needs	Here?	How I know (evidence)
Both triangles are right-angled	✓ **yes**	∠OAP = ∠OBP = 90°, rows 1–2, from Theorem 1
Their hypotenuses are equal	✓ **yes**	both hypotenuses are OP (common)
One other pair of sides is equal	✓ **yes**	OA = OB, radii of the same circle

All three RHS conditions hold, so the triangles are congruent. **Why not SAS?** The known angle (90° at A) lies between OA and AP, and AP is the unknown. SAS needs the angle *between* the two known sides, so it does not apply.

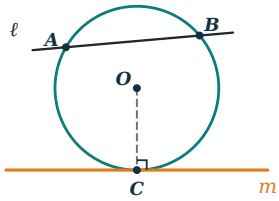
◇ **FIND THE MISSING LINK**

Rows 1 and 2 are the bridge from Theorem 1 to Theorem 2. Remove them and RHS has no right angle to use. That is why Theorem 1 must be proved first.

Doubts 1–2: tangent or secant?

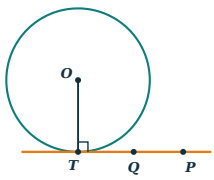
DOUBT 1 *"Two lines in the figure both seem to touch the circle. How do I know which one is the tangent?"*

✗ WHY THIS CONFUSION HAPPENS	A drawing is never exact. A secant that cuts near the edge can look as if it only touches.
◆ UNDERSTAND	Count the common points: a **tangent** has exactly one, a **secant** has exactly two. Decide from the wording (or a proof), never from the look.
★ SPOT THE CLUE	"touches at C", "tangent at C", "point of contact" → tangent. "meets the circle at A and B", "cuts", two named points → secant.
➤ CHOOSE THE THEOREM	No theorem yet: use the **definition**. Only after a line is known to be a tangent may Theorems 1 and 2 be used on it.
✦ SOLVED EXAMPLE	Line ℓ meets the circle at A and B; line m touches it at C. ℓ has two common points, so it is a **secant**. m has one, so it is a **tangent**, and OC $\perp$ m (Theorem 1). Nothing makes OA $\perp$ ℓ.
? WHY THIS STEP?	Classifying first protects every later step: a right angle claimed on a secant is simply false.
⚠ COMMON TRAP	"ℓ looks as if it just grazes the circle, so OA $\perp$ ℓ." Wrong: ℓ meets the circle twice.
✎ YOUR TURN DB1	Line a meets a circle at exactly one point P. Line b passes through P and the centre O. Classify both lines, with reasons.
✓ CHECK YOURSELF	A line through the centre always contains a diameter, so it meets the circle twice.



DOUBT 2 *"What exactly is the point of contact? Is it any point on the tangent?"*

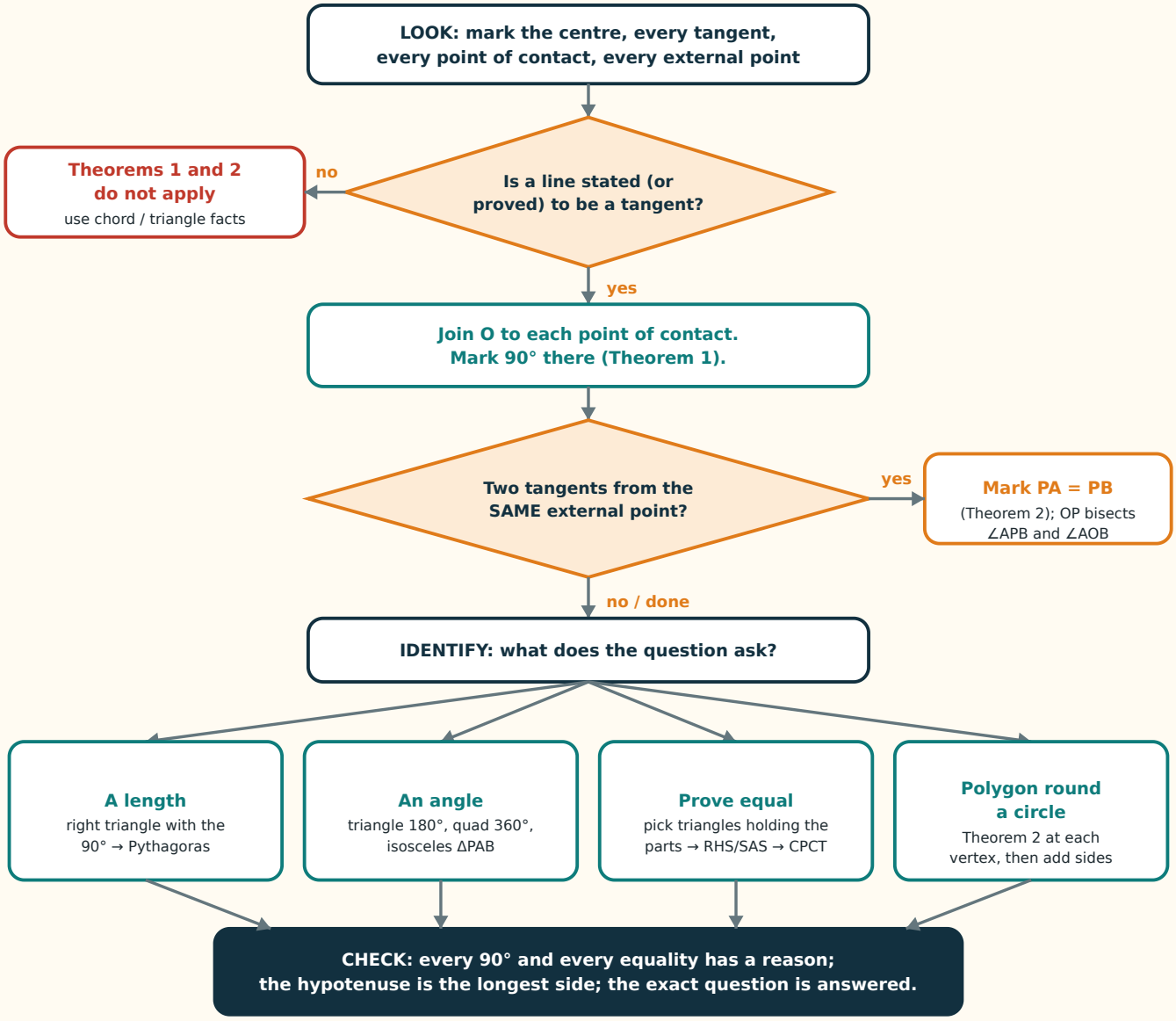
✗ WHY THIS CONFUSION HAPPENS	A tangent line has endless points. It is easy to put the right angle in the wrong place.
◆ UNDERSTAND	The point of contact is the **one** point the tangent shares with the circle. Every other point of the tangent is outside the circle, at distance more than r from O.
★ SPOT THE CLUE	"touches the circle at T": T is the point of contact. In a figure it is where the tangent meets the curve.
➤ CHOOSE THE THEOREM	Theorem 1 puts its 90° **at the point of contact**, between the tangent and the radius to that point.
✦ SOLVED EXAMPLE	PT touches at T and Q is another point of line PT. The right angle is ∠OTP (at T). ∠OQP is **not** 90°: OQ is not a radius (Q is outside the circle).
? WHY THIS STEP?	Finding T first tells you where the right angle and the right triangle are.
⚠ COMMON TRAP	Putting the square at P, the external point, or at Q. The tangent is perpendicular to OT only.
✎ YOUR TURN DB2	XY touches a circle with centre O at A. B is another point on XY. Is OB greater than, equal to, or less than OA? Why?
✓ CHECK YOURSELF	Only one point of the tangent is at distance exactly r from O.



Theorem-selection flowchart

Learning objective: Follow a decision path from the figure to the first correct step.

Follow the arrows from the top. Every branch ends in an action you already know. The skill is choosing the branch, and each diamond is a condition check.



⚠ COMMON TRAP

A flowchart is a guide, not a machine. Many questions go down **more than one** branch: the chord-and-two-tangents problem on page 20 needs a length, an angle bisector and a right triangle. Use the chart to find the first step, then look again.

⚠ COMMON TRAP

Choosing a theorem because the figure “looks like” the textbook figure. Two tangents drawn from *different* points can look exactly like the Theorem 2 picture, and Theorem 2 does not apply to them.

✓ CHECK YOURSELF

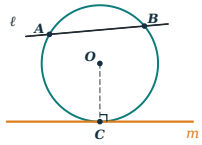
Try the chart now. “PA and PB are tangents from P; ∠AOB = 116°; find ∠APB.” Which boxes do you pass through? (Answer: tangent → join radii, 90° → same external point → an angle → quadrilateral 360°, giving 64°.)

Mistake clinic (1 of 3)

MISTAKE 1 · Treating a secant as a tangent

✗ STUDENT'S WORK

"Line l meets the circle at A and B. OA is a radius, so OA ⊥ l (radius ⊥ tangent)."

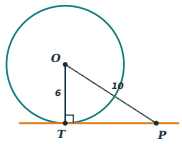


⚠ WHERE IT FAILS	*l* has two common points, so it is a secant. Theorem 1 is about tangents only.
? WHY TEMPTING	The words "radius" and "line" are both there, and the theorem is fresh in memory.
✓ CORRECTED	*l* is a secant; no right angle at A is known.
➤ PREVENTION RULE	**Before using Theorem 1, count the common points: exactly one.**
✎ FIX IT E1	Line *m* passes through P on a circle and through Q inside it. A student writes OP ⊥ *m*. Explain the error.

MISTAKE 2 · Right angle at the wrong vertex, wrong hypotenuse

✗ STUDENT'S WORK

"PT touches at T; OT = 6, OP = 10. ∠OPT = 90°, so $OT^2 = OP^2 + PT^2$, $PT^2 = 36 - 100$."

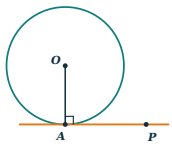


⚠ WHERE IT FAILS	The right angle is at T, the point of contact, so the hypotenuse is OP, not OT.
? WHY TEMPTING	The 90° sign is placed at the letter P that appears in the question.
✓ CORRECTED	∠OTP = 90° (Theorem 1). $PT^2 = OP^2 - OT^2 = 100 - 36 = 64$, PT = 8.
➤ PREVENTION RULE	**Right angle at the point of contact; hypotenuse runs from O to the external point.**
✎ FIX IT E2	PA touches a circle at A. OA = 10 cm, OP = 26 cm. A student writes $PA^2 = OA^2 + OP^2$. Correct it and find PA.

MISTAKE 3 · Using an arbitrary radius

✗ STUDENT'S WORK

"The tangent at A … OB is a radius, so OB ⊥ the tangent."



⚠ WHERE IT FAILS	B is not the point of contact. Only OA is perpendicular to the tangent at A.
? WHY TEMPTING	"All radii are equal" is stretched into "all radii behave the same".
✓ CORRECTED	OA ⊥ tangent at A. For angles at B, use ΔOAB with OA = OB.
➤ PREVENTION RULE	**Follow the radius to its end point: is that the point of contact?**
✎ FIX IT E3	The tangent at C meets a circle with centre O; D is another point of the circle. Which of OC, OD is ⊥ to the tangent? Why?

Practice 1 · Recognise

Learning objective: Identify tangents, secants, points of contact and the clues that point to each theorem.

1 Recognise | 2 Apply | 3 Connect | 4 Board-style

Answers: Section H

P1 A line meets a circle at exactly two points. The line is called
(A) a tangent (B) a secant (C) a radius (D) a non-intersecting line

Purpose: Recognise a secant. **easy ★☆☆**

Working space (use separate paper if needed)

P2 How many tangents can be drawn to a circle through a point that lies on the circle?
(A) 0 (B) 1 (C) 2 (D) infinitely many

Purpose: Count tangents from a position. **easy ★☆☆**

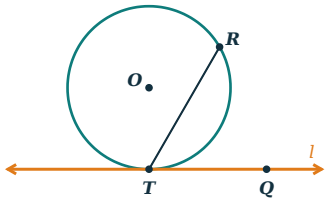
Working space (use separate paper if needed)

P3 Fill in the blanks. (a) A tangent and its circle have ____ common point(s). (b) That common point is called the ____. (c) Tangents at the two ends of a diameter are ____. (d) At most ____ tangents of a circle can be parallel to a given line.

Purpose: Use the vocabulary precisely. **easy ★☆☆**

Working space (use separate paper if needed)

P4 Line *l* touches the circle (centre O) at T; Q is on *l*; R is on the circle. (a) Name the point of contact. (b) Which radius is perpendicular to *l*? (c) Is ∠OQT = 90°? (d) Is TR a tangent? Give a reason for each.



Purpose: Read a diagram with reasons. **easy ★☆☆**

Working space (use separate paper if needed)

P5 A circle has radius 7 cm. How many tangents to it pass through a point at distance (a) 5 cm, (b) 7 cm, (c) 9.5 cm from the centre?

Purpose: Apply the position test. **easy ★☆☆**

Working space (use separate paper if needed)

P6 True or false? Give a reason. (a) Every line through a point outside a circle is a tangent. (b) The radius to any point of a circle is perpendicular to every tangent. (c) The two tangents from an external point are equal in length. (d) A tangent can pass through the centre.

Purpose: Spot over-generalisations. **moderate ★★☆**

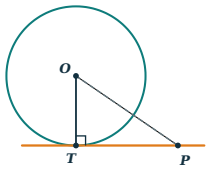
Working space (use separate paper if needed)

One-page revision sheet

Learning objective: Recall every definition, both theorems with their conditions, and the clues that point to them.

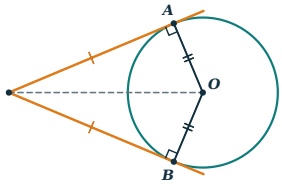
◆ UNDERSTAND

Tangent: a line meeting the circle at exactly one point, the **point of contact**. **Secant:** a line meeting it at two points. **Length of a tangent** from an external point P: PT, from P to the point of contact. **Tangents through P:** inside 0 · on the circle 1 · outside 2 (test: compare OP with r).



Theorem 1 CORE · PROVE

The tangent at any point of a circle is ⊥ to the radius through the point of contact.
Needs: a tangent; the radius to *its* contact point.
Proof idea: every other point of the tangent is outside, so OP is the shortest distance, hence ⊥.



Theorem 2 CORE · PROVE

Tangents from an external point are equal: PA = PB.
Needs: both tangents from the same point.
Proof: 90° at A, B (Thm 1); OA = OB; OP common; RHS; CPCT.

Diagram-reading clues

If you see…	…do this
“touches”, “tangent at T”, “point of contact”	join O to T; mark 90° (Theorem 1)
tangent + centre + a length	right triangle; hypotenuse = O to external point
two tangents from one point P	PA = PB; OP bisects ∠APB and ∠AOB
angle at P or at O between two tangents	∠APB + ∠AOB = 180°
polygon around a circle	two equal tangents at every vertex
chord of a big circle touching a smaller concentric one	point of contact is the chord’s midpoint

★ THE FOUR-QUESTION CHECK — before you write any step

Condition the theorem needs	Here?	How I know (evidence)
Is the line really a tangent?	☐	stated or proved, never “it looks like it”
Does the radius end at its point of contact?	☐	otherwise there is no right angle
Do both tangent segments start at one point?	☐	otherwise there is no equality
Is each 90° and each equality backed by a reason?	☐	given, a theorem, or proved

Four ticks → use the theorem. One “no” → choose a different fact.

⚠ COMMON TRAP

Final error checklist. ☐ secant treated as tangent (E1) ☐ right angle at the wrong vertex (E2) ☐ arbitrary radius (E3) ☐ 90° assumed (E4) ☐ tangents from different points (E5) ☐ SAS instead of RHS (E6) ☐ missing reasons (E7) ☐ right number, missing reasons (E8) ☐ measuring the figure (E9)

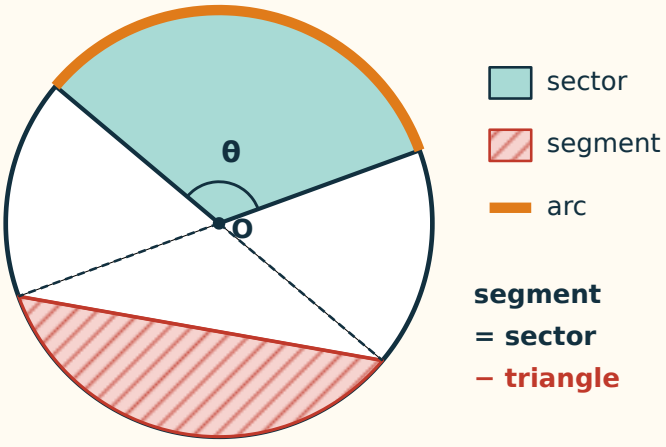
MATHS WITHOUT GUESSWORK

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Understand the Diagram. Choose the Formula. Solve Independently.

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Spot the region first. Then decide what to subtract or combine.

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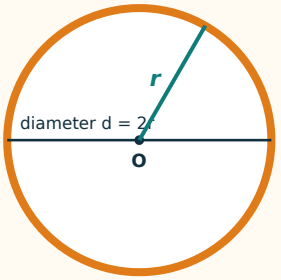
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Radius, diameter, circumference and area

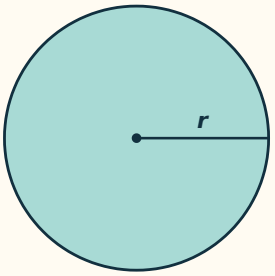
PREREQUISITE

Learning objective: Find the radius from any given measurement, and never confuse a boundary length with an area.



Circumference = 2πr (a length)

boundary vs inside



Area = πr² (a region)

◆ UNDERSTAND

The **radius** r joins the centre to the circle. The **diameter** d passes through the centre, so **d = 2r** and **r = d ÷ 2**. The **circumference** is the length of the boundary: C = 2πr = πd. The **area** is the amount of flat surface inside: A = πr².

➤ TWO DIFFERENT QUANTITIES

Small example (r = 7 cm, π = $\frac{22}{7}$):

C = 2 × $\frac{22}{7}$ × 7 = **44 cm**

A = $\frac{22}{7}$ × 7 × 7 = **154 cm²**

★ SPOT THE CLUE

Hidden radius. "Diameter 21 cm" → r = 10.5 cm. "Circumference 44 cm" → 2πr = 44 → r = 7 cm. "Largest circle in a square of side 12 cm" → the diameter equals the side, so r = 6 cm.

? WHY THIS STEP?

Why is the radius squared? Area counts unit squares, so it must be "length × length". Doubling the radius makes the circle twice as wide *and* twice as tall, so the area becomes 2 × 2 = 4 times as big. Circumference is a single length, so it only doubles.

⚠ COMMON TRAP

Using the diameter as r. With d = 14 cm, writing π × 14² gives 616 cm², four times too big. Always write "r = ..." as your first line.

If the question gives…	then r =	Example
the diameter d	d ÷ 2	d = 21 cm → r = 10.5 cm
the circumference C	C ÷ (2π)	C = 88 cm → r = 88 × 7 ÷ 44 = 14 cm
the area A	√(A ÷ π)	A = 154 cm² → r² = 49 → r = 7 cm
"largest circle in a square of side a"	a ÷ 2	a = 12 cm → r = 6 cm

✓ CHECK YOURSELF

C1. A circle has a diameter of 28 cm. Find (a) its circumference and (b) its area. Write the unit of each.

Method 2: area of a sector

CORE

Learning objective: Find a sector's area as a fraction of the whole circle, and explain why the formula works.

Hold this question: *If 360° gives the whole area πr², what does 1° give? What does θ give?*

◆ UNDERSTAND

Unitary method. The whole circle is a "sector" of angle 360°, with area πr². At 360° → πr². At 1° → $\frac{\pi r^2}{360}$. At θ → $\frac{\theta}{360} \times \pi r^2$.

➤ CHOOSE THE FORMULA

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

θ = central angle in degrees, r = radius. "The sector is $\frac{\theta}{360}$ of the whole circle."

SPOT THE REGION → SUBTRACT OR COMBINE

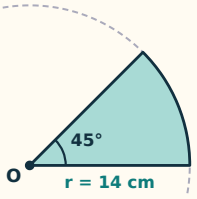
1 · SHADED REGION	A sector with central angle θ.
2 · FAMILIAR WHOLE	The full circle of the same radius.
3 · SUBTRACT / COMBINE	Take the fraction θ/360 of it (nothing removed yet).
REGION EQUATION	**sector = (θ/360) × circle**

WORKED EXAMPLE **r = 14 cm, θ = 45°**

Fraction = $\frac{45}{360} = \frac{1}{8}$

Area of circle = $\frac{22}{7} \times 14 \times 14 = 616$ cm²

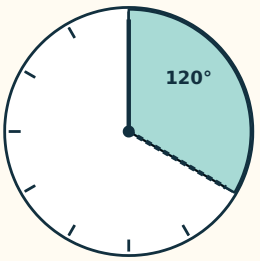
Area of sector = ⅛ × 616 = **77 cm²**



WORKED EXAMPLE **Minute hand 21 cm long, 20 minutes**

Angle turned = 20 × 6° = 120°, fraction = ⅓

Area swept = ⅓ × $\frac{22}{7}$ × 21 × 21 = ⅓ × 1386 = **462 cm²**



XII → IV: 20 minutes

? WHY THIS STEP?

Why find the whole circle first? It makes the fraction idea visible and keeps numbers simple: ⅛ of 616 is easier than multiplying four numbers together.

⚠ COMMON TRAP

Using the minutes as the angle. 20 minutes is not 20°. Convert first: 6° per minute.

✎ YOUR TURN

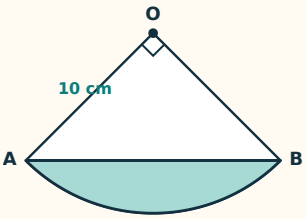
T2. The minute hand of a clock is 7 cm long. Find the area it sweeps in 15 minutes.

Method 4 (part 2): area of a segment, 90° and 60°

CORE

Learning objective: Find a segment as sector − triangle, writing the region equation first.

Hold this question: *The shaded region has a straight edge and a curved edge. What is the straight edge?*



SPOT THE REGION → SUBTRACT OR COMBINE

1 · SHADED REGION	Between chord AB and arc AB: a minor segment.
2 · FAMILIAR WHOLE	Sector OAB (two radii + arc).
3 · SUBTRACT / COMBINE	Subtract triangle OAB.
REGION EQUATION	**segment = sector OAB − ΔOAB**



WORKED EXAMPLE **r = 10 cm, ∠AOB = 90°, π = 3.14**

Sector = $\frac{90}{360} \times 3.14 \times 10 \times 10 = \frac{1}{4} \times 314 = 78.5$ cm²

ΔOAB = ½ × OA × OB = ½ × 10 × 10 = 50 cm² (OA ⊥ OB)

Segment = 78.5 − 50 = **28.5 cm²**

? WHY THIS STEP?

Why is ½ × OA × OB allowed? At 90°, OA is perpendicular to OB, so OB is the height when OA is the base. No other angle gives you this shortcut.

WORKED EXAMPLE **r = 21 cm, ∠AOB = 60°, √3 = 1.73**

Sector = $\frac{60}{360} \times \frac{22}{7} \times 21 \times 21 = 231$ cm²

ΔOAB is equilateral (side 21): $\frac{\sqrt{3}}{4} \times 21^2 = \frac{441\sqrt{3}}{4} = \frac{441 \times 1.73}{4} = 190.7325$ cm²

Segment = 231 − 190.7325 = 40.2675 ≈ **40.27 cm²**

(Exact form, if asked: $(231 - \frac{441\sqrt{3}}{4})$ cm².)

✔ CHECK YOURSELF

Is it sensible? A 60° segment is a thin sliver: 40.27 is about 17% of the sector (231). A segment can never be larger than its sector.

⚠ COMMON TRAP

Stopping at the sector. If the diagram shows a chord, the triangle must be removed. "Sector area" is not the answer to a segment question.

✎ YOUR TURN

T6. A chord of a circle of radius 14 cm subtends a right angle at the centre. Find the area of the minor segment.

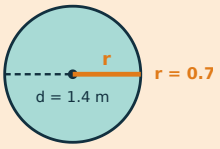
Doubts 1 and 2: radius and "which quantity?"

DOUBT 1 *"The formula says πr², but the question gave me the diameter. Why can't I just use that number?"*

✗ WHY IT HAPPENS
The formula uses whatever letter is in it, and students substitute whatever length the question offers.

◆ THE CONCEPT
r is the distance from the **centre** to the circle. πr² counts about 3.14 squares of side r. Squares of side d = 2r are four times larger, so using d makes the answer four times too big.

★ WORKED EXAMPLE
A circular table top has diameter 1.4 m. r = 1.4 ÷ 2 = 0.7 m. Area = $\frac{22}{7}$ × 0.7 × 0.7 = **1.54 m²**.



? WHY THIS STEP?
Halving first is forced by the definition of r. Writing "r = 0.7 m" as a separate line shows the examiner you know it.

⚠ WRONG APPROACH
$\frac{22}{7}$ × 1.4 × 1.4 = 6.16 m², exactly 4 times too much.

✎ YOUR TURN DB1
A circle has diameter 42 cm. Find its area.

✓ REMEMBER
Write **"r = ..."** before every formula.

DOUBT 2 *"Some questions say "how much wire" and some say "how much cloth". How do I know whether to find area or circumference?"*

✗ WHY IT HAPPENS
Both formulas contain π and r, so students react to the *circle* instead of to the *question*.

◆ THE CONCEPT
Ask: could I lay the answer along a line (a **length**: wire, fence, rim, border) or spread it over a surface (an **area**: cloth, paint, grass)? Lengths use 2πr; areas use πr².

★ WORKED EXAMPLE
Circular park, r = 21 m. Fencing = 2 × $\frac{22}{7}$ × 21 = **132 m**. Grass to cover it = $\frac{22}{7}$ × 21 × 21 = **1386 m²**.



? WHY THIS STEP?
The fence goes round the boundary only; the grass covers every part of the surface inside.

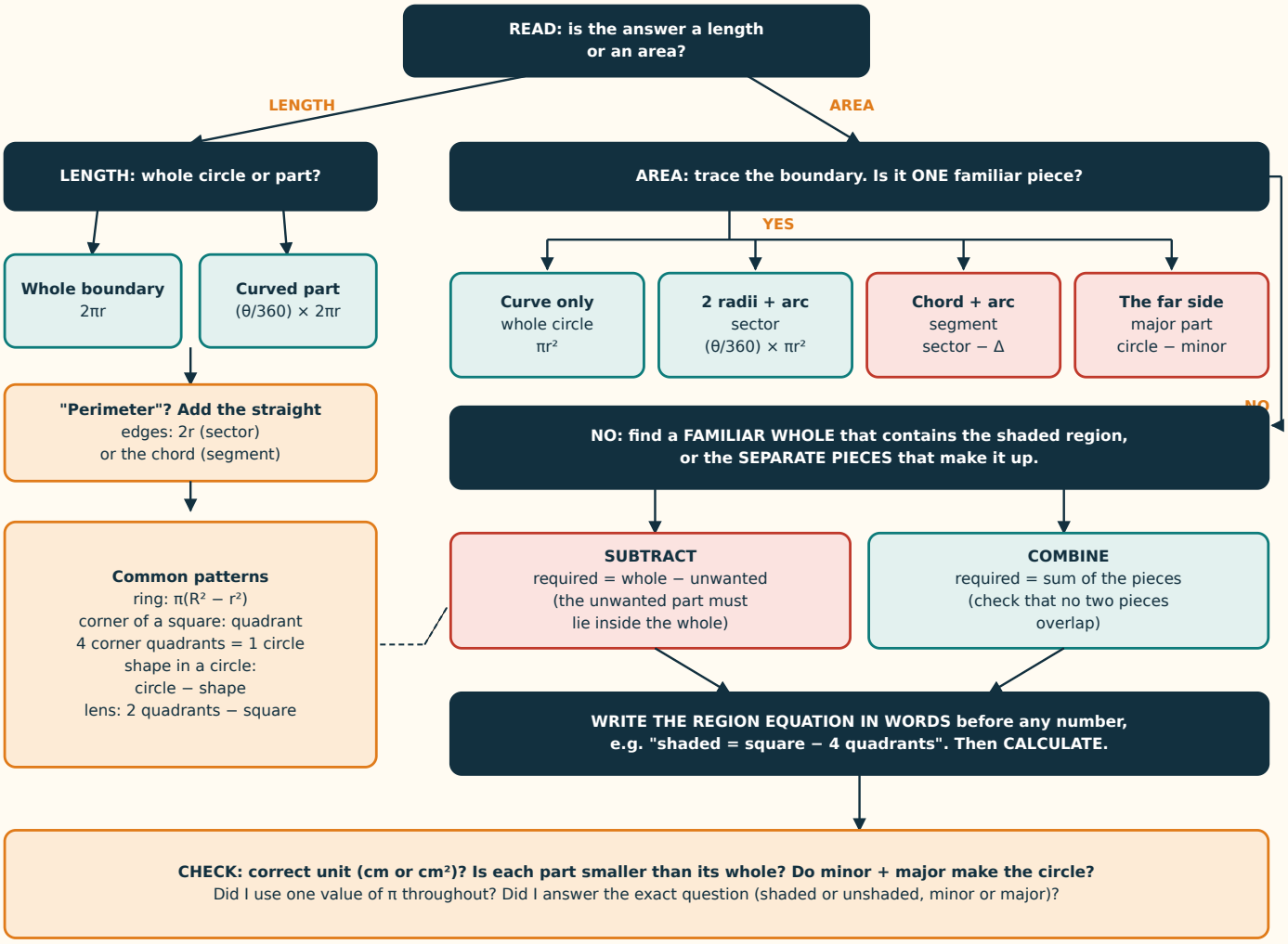
⚠ WRONG APPROACH
Answering "1386 m of fencing". The unit would be m², which cannot measure a fence.

✎ YOUR TURN DB2
A circular park has radius 35 m. Find (a) the length of fencing around it, (b) the area to be planted with grass.

✓ REMEMBER
Length → cm, m. Area → cm², m². The unit tells you which formula.

Decision tree: what should I calculate first?

Learning objective: Follow the branches from the question to your first calculation, and know what to check at the end.



Navy = a question to ask · teal = a single formula · coral = subtract · orange = reminders and checks

★ SPOT THE CLUE

Run the tree once. "A chord of a circle of radius 14 cm subtends 90° at the centre. Find the area of the minor segment." → AREA → one familiar piece? Yes: chord + arc → **segment = sector − ΔOAB** → write it in words → 154 − 98 = 56 cm² → CHECK: cm² ✓, 56 < 154 ✓.

? WHY THIS STEP?

Why does the tree start with "length or area"? It instantly halves the choices and fixes the unit of your answer. Students who skip it are the ones who give a fence in m².

⚠ COMMON TRAP

A tree is a guide, not a machine. Some questions need both branches: "find the area of the segment and the length of its boundary". Run the tree once for each part.

Mistake clinic (1 of 3): reading the circle

MISTAKE 1 · Diameter used as the radius

✗ STUDENT'S WORK	*Area of a circle of diameter 14 cm.* "$\frac{22}{7} \times 14 \times 14 = 616$ cm²"
⚠ WHERE IT FAILS	14 cm is the diameter. The formula needs r = 7 cm, so the answer is 4 times too big.
✓ CORRECTED	r = 14 ÷ 2 = 7 cm. Area = $\frac{22}{7} \times 7 \times 7 = 154$ cm².
➤ PREVENTION RULE	Write "r = …" as your first line.
✎ FIX IT E1	Find the area of a circle of diameter 35 cm.

MISTAKE 2 · Circumference used instead of area

✗ STUDENT'S WORK	*Area of a circle of radius 7 cm.* "$2 \times \frac{22}{7} \times 7 = 44$ cm²"
⚠ WHERE IT FAILS	2πr is the boundary length (in cm), not the surface inside. A length cannot be measured in cm².
✓ CORRECTED	Area = πr² = $\frac{22}{7} \times 7 \times 7 = 154$ cm².
➤ PREVENTION RULE	An area formula always contains r² and gives square units.
✎ FIX IT E2	Find the area of a circle of radius 21 cm.

MISTAKE 3 · Radius not squared

✗ STUDENT'S WORK	*Sector of radius 14 cm, angle 90°.* "¼ × $\frac{22}{7} \times 14 = 11$ cm²"
⚠ WHERE IT FAILS	The radius appears only once, so this multiplies a length by a number: it cannot be an area.
✓ CORRECTED	¼ × $\frac{22}{7} \times 14 \times 14 = ¼ \times 616 = 154$ cm².
➤ PREVENTION RULE	Write r × r in full rather than r².
✎ FIX IT E3	Find the area of a sector of radius 7 cm and angle 60°.

MISTAKE 4 · Wrong central angle

✗ STUDENT'S WORK	*Area swept by a 21 cm minute hand in 10 minutes.* "(10/360) × 1386 = 38.5 cm²"
⚠ WHERE IT FAILS	10 minutes is not 10°. The minute hand turns 6° every minute.
✓ CORRECTED	Angle = 10 × 6° = 60°. Area = ⅙ × 1386 = 231 cm².
➤ PREVENTION RULE	Convert time to degrees first: 6° per minute.
✎ FIX IT E4	Find the area swept by a minute hand 14 cm long in 35 minutes.

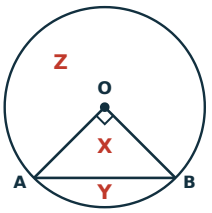
Practice 1: recognise

Learning objective: Name regions, choose the quantity, and find the radius and angle before calculating anything.

1 Recognise | 2 Apply | 3 Connect | 4 Board-style

No calculation needed for most. Answers: Section H.

P1 In the figure, O is the centre and ∠AOB = 90°. X is ΔOAB, Y is the region between chord AB and the minor arc, and Z is the rest of the circle. Name: (a) X, (b) Y, (c) X and Y together, (d) Z, (e) X and Z together.



Purpose: Name regions by their boundaries. **easy ★☆☆**

Working space (use separate paper if needed)

P2 Length or area? Say which, and which formula: (a) wire for a circular photo frame; (b) cloth for a round tablecloth; (c) the distance moved by the tip of a minute hand in 15 minutes; (d) the region cleaned by a wiper; (e) fencing for a semicircular garden.

Purpose: Separate boundary questions from region questions. **easy ★☆☆**

Working space (use separate paper if needed)

P3 Find the radius: (a) diameter 18 cm; (b) circumference 66 cm; (c) the largest circle inside a square of side 12 cm; (d) the circle passing through all four corners of a square of side 10 cm.

Purpose: Find hidden radii. **moderate ★★☆**

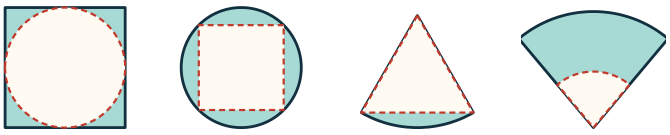
Working space (use separate paper if needed)

P4 Find the angle at the centre: (a) a minute hand in 25 minutes; (b) an hour hand in 2 hours; (c) each of 10 equal sectors; (d) the major sector when the minor sector is 75°.

Purpose: Turn situations into central angles. **easy ★☆☆**

Working space (use separate paper if needed)

P5 Write the region equation in words for each shaded (teal) region. No numbers.



(a) (b) (c) (d)

Purpose: Spot the region, then subtract. **moderate ★★☆**

Working space (use separate paper if needed)

P6 True or false? Give a reason. (a) The area of a semicircle is ½πr². (b) The perimeter of a semicircle is πr. (c) Segment = sector + triangle. (d) Doubling the radius doubles the area.

Purpose: Catch common misconceptions. **moderate ★★☆**

Working space (use separate paper if needed)

Revision sheet

Learning objective: Recall every formula, triangle and region pattern from one page.

Quantity	Formula	Remember
Circle	$A = \pi r^2$, $C = 2\pi r = \pi d$	Find r first: $r = d \div 2$.
Sector (angle θ)	$(\theta/360) \times \pi r^2$	Two radii + arc. Semicircle $\frac{1}{2}\pi r^2$, quadrant $\frac{1}{4}\pi r^2$.
Arc	$(\theta/360) \times 2\pi r$	A length, in cm.
Perimeter	sector: arc + 2r; segment: arc + chord	Include every straight edge.
Segment (minor)	sector − ΔOAB	Syllabus angles only: 60°, 90°, 120°.
Major sector / segment	πr^2 − minor part	Or $(360° - \theta)/360 \times \pi r^2$ for the major sector.

➤ THE THREE TRIANGLES

∠AOB	chord AB	area ΔOAB
60°	r	$(\sqrt{3}/4)\ r^2$
90°	$r\sqrt{2}$	$\frac{1}{2}\ r^2$
120°	$r\sqrt{3}$	$(\sqrt{3}/4)\ r^2$

◆ VALUES AND UNITS

π: $\frac{22}{7}$ unless told; 3.14 if told; keep π if "in terms of π".

Clocks: minute hand 6°/min; hour hand 30°/hour. **Units:** lengths in cm, areas in cm^2; 1 m^2 = 10 000 cm^2. **Rounding:** only at the end.

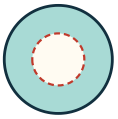
THE EIGHT REGION PATTERNS



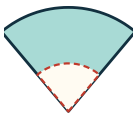
Segment
sector − triangle



Major sector
circle − minor sector



Ring
$\pi(R^2 - r^2)$



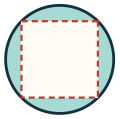
Part of a ring
big sector − small sector



Corners
square − circle



Corner quadrants
square − 4 quadrants



Around a square
circle − square



Lens
2 quadrants − square

In every case: ① spot the shaded region ② name the familiar whole (or pieces) ③ subtract the unwanted part, or add the pieces without overlap ④ write the region equation in words, **then** calculate.

1 READ | 2 IDENTIFY | 3 VISUALISE | 4 CHOOSE | 5 CALCULATE | 6 SUBTRACT / COMBINE | 7 CHECK

⚠ COMMON TRAP

Final error checklist. ☐ d used as r (E1) ☐ 2πr for area (E2) ☐ r not squared (E3) ☐ minutes as degrees (E4) ☐ sector given as segment (E5) ☐ added a hole (E6) ☐ wrong whole (E7) ☐ mixed units (E8) ☐ missing ² (E9) ☐ early rounding (E10) ☐ wrong π (E11) ☐ skipped steps (E12)

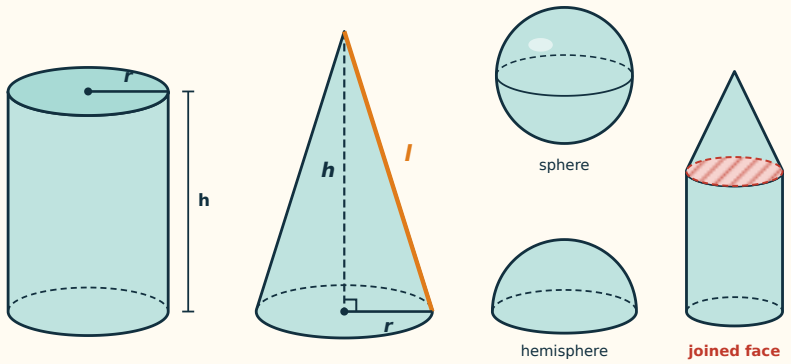
MATHS WITHOUT GUESSWORK

Surface Areas and Volumes

Identify the Solid.
Choose the Formula.
Solve Independently.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27



Knowing a formula is not enough. Choosing the right one is the skill.

Core topics: cylinder · cone · sphere · hemisphere · cuboid · surface areas and volumes of combinations of two solids

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher. Diagrams are not drawn to scale.

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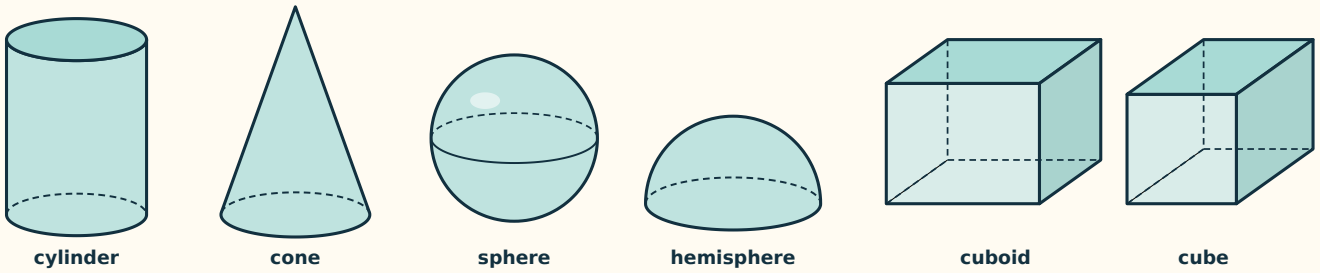
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Flat shapes and solids

PREREQUISITE

Learning objective: Describe each solid by its faces: which are flat, which are curved, and how many there are.



Curved surfaces are pale teal; flat faces are darker teal. Dashed lines are hidden edges at the back.

◆ UNDERSTAND

A **circle** or a **rectangle** is flat (2-D): it has only area. A **solid** (3-D) has length, breadth and height. It has **surfaces**, which have area, and it encloses **space**, which has volume.

Every solid in this chapter is built from two kinds of surface: **flat faces** (circles, rectangles, squares) and **curved surfaces** (the side of a can, the outside of a ball).

Solid	Everyday object	Flat faces	Curved surfaces	Model limits
Cylinder	tin can, pipe, pillar	2 circles (if closed)	1	A real can has a rim and thin walls
Cone	ice-cream cone, tent top	1 circle	1	A real cone may have a rounded tip
Sphere	ball, marble	0	1	A football has seams and air inside
Hemisphere	bowl, dome	1 circle	1	A bowl is hollow and open
Cuboid / cube	box, brick, die	6 rectangles / squares	0	Edges may be rounded

? WHY THIS STEP?

Why count faces? Because surface-area formulas are just sums of face areas. If you know a closed cylinder has one curved surface and two circles, you can *build* its TSA instead of remembering it: $2\pi rh + \pi r^2 + \pi r^2$.

⚠ COMMON TRAP

"A sphere and a circle are the same shape." A circle is flat and has an area πr^2. A sphere is a solid ball: its *surface* is $4\pi r^2$ and it encloses a *volume* $\frac{4}{3}\pi r^3$.

✓ CHECK YOURSELF

C1. How many flat faces and how many curved surfaces does each have? (a) a closed cylinder (b) a cylinder open at the top (c) a pipe open at both ends (d) a solid hemisphere (e) a sphere.

Concept 1: the cylinder (surfaces)

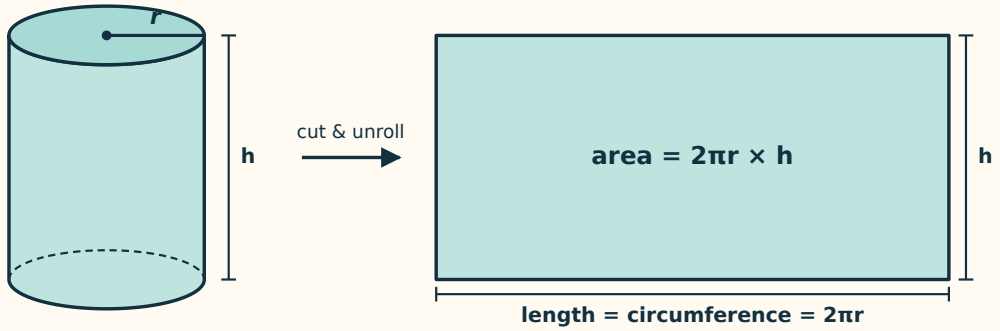
CORE

Learning objective: Explain why CSA = 2πrh, and build the TSA of a closed cylinder from its faces.

Hold this question: *A label wraps exactly once round a tin can. Why does its length equal the circumference of the can?*

◆ UNDERSTAND

Object first. A tin can, a pipe, a pillar. A **right circular cylinder** has two equal circular ends (radius r) joined by a curved surface, with height h at right angles to the ends.



➤ CHOOSE THE FORMULA

CSA = 2πrh **area of one end = πr²** **TSA (closed) = 2πrh + 2πr² = 2πr(r + h)**

WORKED EXAMPLE **Closed cylinder, r = 7 cm, h = 10 cm**

CSA = $2 \times \frac{22}{7} \times 7 \times 10 =$ **440 cm²**

Two ends = $2 \times \frac{22}{7} \times 7 \times 7 = 308$ cm²

TSA = 440 + 308 = **748 cm²** (or 2πr(r + h) = 44 × 17 = 748 ✓)

? WHY THIS STEP?

Why the circumference? The label goes round the can exactly once, so its length is the distance round the circular end: 2πr. Its width is the height h. A rectangle's area is length × width, so CSA = 2πr × h.

★ SPOT THE CLUE

Use CSA for: label, side wall, pillar painted (top and bottom not), pipe open at both ends. **Use TSA** only if both ends are part of the surface asked for.

⚠ COMMON TRAP

2πr² for the ends. That is right only for *two* ends. An open container has **one** end: πr².

✎ YOUR TURN

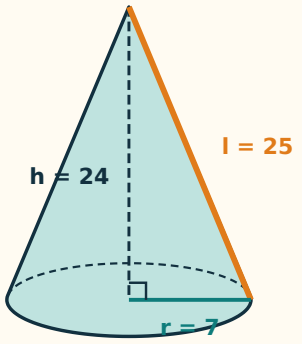
T1. A closed cylindrical tin has radius 3.5 cm and height 10 cm. Find (a) the area of its label (curved surface) and (b) the area of tin sheet needed to make it.

Concept 2: the cone and its two heights

CORE

Learning objective: Identify which height a question gives and which height each cone formula needs.

Hold this question: *A cone has slant height 25 cm and radius 7 cm. Can I put 25 into the volume formula?*



◆ UNDERSTAND

Object first. An ice-cream cone, a birthday cap, the top of a tent. A **right circular cone** has a circular base and an apex directly above the centre of the base.

It has **two heights**: the perpendicular height **h** (inside the cone, centre to apex) and the slant height **l** (on the surface, edge to apex). They are linked by $\mathbf{l^2 = r^2 + h^2}$.

Formula	Needs	Why that height?
CSA = πrl	slant height l	The curved surface lies along the slope; when unrolled it is a sector whose radius is l.
V = ⅓πr²h	perpendicular height h	Volume measures how high the space rises above the base, straight up.

WORKED EXAMPLE **r = 7 cm, h = 24 cm: find l**

$l^2 = r^2 + h^2 = 49 + 576 = 625$

$l =$ **25 cm**

WORKED EXAMPLE **r = 7 cm, l = 25 cm: find h**

$h^2 = l^2 - r^2 = 625 - 49 = 576$

$h =$ **24 cm**

→ PREDICT THE NEXT STEP

A question gives r and l and asks for the **volume**. What must you calculate *before* using ⅓πr²h? (Answer: h, from $h^2 = l^2 - r^2$.)

⚠ COMMON TRAP

Swapping heights. ⅓π × 49 × 25 uses l in the volume formula. The true volume (with h = 24) is 1232 cm³; with 25 you get 1283.33 cm³, which is wrong.

✎ YOUR TURN

T3. A cone has radius 9 cm and slant height 15 cm. (a) Find its perpendicular height. (b) Which of the two heights would you use for its CSA, and which for its volume?

Doubts 1 and 2: area or volume? CSA or TSA?

DOUBT 1 *"How do I know whether the question asks for surface area or volume?"*

✗ WHY THIS CONFUSION HAPPENS	Both come from the same measurements of the same solid, so students react to the solid, not to the question.
◆ UNDERSTAND	Surface area measures **covering** (paint, paper, sheet, canvas). Volume measures **filling or occupying** (water, air, sand, metal, capacity).
★ SPOT THE CLUE	Covering verbs: paint, wrap, polish, cover, canvas, sheet. Filling words: hold, contain, capacity, litres, melt, how much metal.
➤ CHOOSE THE FORMULA	Covering → an area formula (cm^2). Filling → a volume formula (cm^3 or litres).
✎ SOLVED EXAMPLE	A closed cubical gift box has edge 10 cm. Paper to wrap it (no overlap) = 6 × 10 × 10 = **600 cm^2**. Space inside = 10^3 = **1000 cm^3**.
? WHY THIS STEP?	Wrapping touches every face, so all six face areas are added. Filling counts unit cubes, so three edges are multiplied.
⚠ COMMON TRAP	Giving 1000 cm^2 of paper because "it is the bigger number". The unit cm^2 cannot measure space.
✎ YOUR TURN DB1	A closed cylindrical tin has r = 7 cm and h = 10 cm. Find (a) the tin sheet needed to make it and (b) how much it holds.
✓ CHECK YOURSELF	One answer is in cm^2, the other in cm^3. Check: the sheet answer must be more than the CSA alone.

DOUBT 2 *"What is the real difference between CSA and TSA?"*

✗ WHY THIS CONFUSION HAPPENS	Both are "surface area", and for a sphere there is no difference, so students think there never is.
◆ UNDERSTAND	CSA is only the curved part. TSA is every surface: curved **plus** flat faces. TSA − CSA = the flat faces.
★ SPOT THE CLUE	"Curved", "lateral", "side", "label" → CSA. "Total", "whole surface", "closed", "solid … all over" → TSA.
➤ CHOOSE THE FORMULA	Cylinder: CSA $2\pi rh$, TSA $2\pi r(r + h)$. Cone: CSA πrl, TSA $\pi r(l + r)$. Hemisphere: CSA $2\pi r^2$, TSA $3\pi r^2$.
✎ SOLVED EXAMPLE	Closed cylinder, r = 7 cm, h = 10 cm: CSA = 2 × 22/7 × 7 × 10 = **440 cm^2**; TSA = 440 + 2 × 154 = **748 cm^2**.
? WHY THIS STEP?	The two ends are separate circles of area πr^2 = 154 cm^2 each, added only for TSA.
⚠ COMMON TRAP	Using TSA for a label, which would also cover the top and bottom of the tin.
✎ YOUR TURN DB2	A cone has r = 7 cm and slant height 10 cm. Find its CSA and its TSA.
✓ CHECK YOURSELF	TSA − CSA should equal one circle of radius 7 cm.

Question clue → solid → quantity → formula

Learning objective: Translate the words of a question into a solid, a quantity and a first step.

Question clue	Solid	Quantity	Formula	First step
"Canvas for a conical tent"	cone (no floor)	curved surface	πrl	find l if h is given
"Tin to make a closed can"	closed cylinder	TSA	$2\pi r(r + h)$	halve d; check both ends exist
"Water a glass can hold"	cylinder	capacity (volume)	$\pi r^2 h$	use inner r; convert to litres
"Paint the outside of a bowl"	hemisphere (open)	outer CSA	$2\pi r^2$	no flat face: it is open
"Polish a solid hemisphere all over"	solid hemisphere	TSA	$3\pi r^2$	include the flat face
"Leather to make a football"	sphere	surface area	$4\pi r^2$	halve d
"Metal in a solid ball"	sphere	volume	$\frac{4}{3}\pi r^3$	r cubed; cm^3
"Sand in a conical heap"	cone	volume	$\frac{1}{3}\pi r^2 h$	need h, not l
"Paint a toy: cone on a hemisphere"	combination	exposed surface	$\pi rl + 2\pi r^2$	joined circle not painted
"Air inside a tent: cylinder + cone"	combination	total volume	$\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$	add the two volumes
"Wood left after a cone is scooped out of a cylinder"	combination (removed)	volume	$\pi r^2 h - \frac{1}{3}\pi r^2 h$	subtract the removed part
"Ribbon round the rim of a can"	circle (edge)	a length	$2\pi r$	not an area at all

➤ WHY THIS FORMULA?

Read the table from left to right, never right to left. The formula is the *result* of the three columns before it. Two rows can share a solid (sphere: leather vs metal) and still need different formulas, because the quantity is different.

⚠ COMMON TRAP

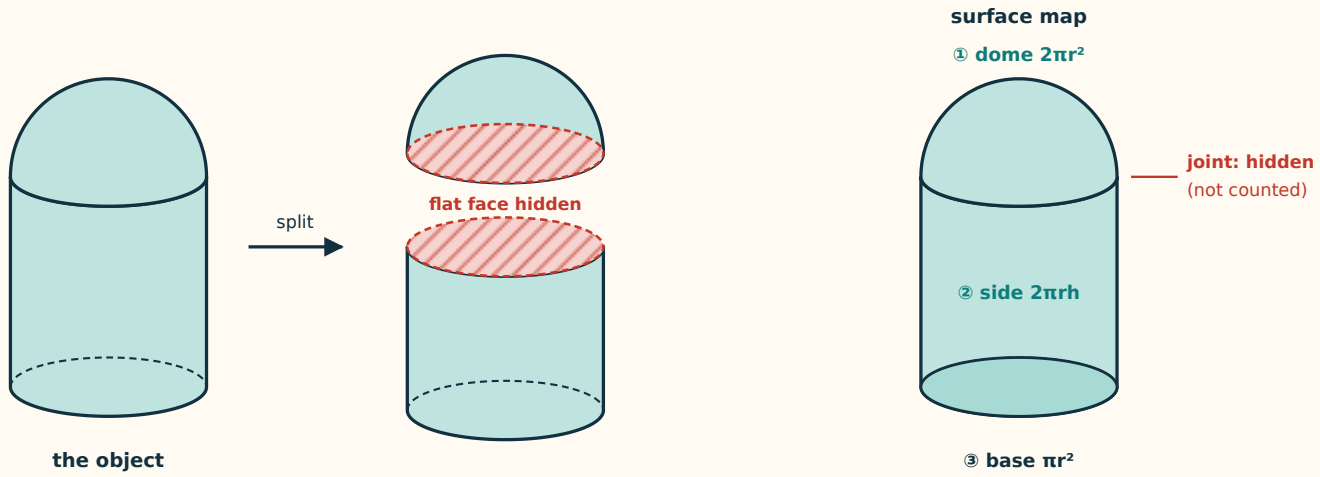
Same clue, different object. "Paint the dome" (open, CSA $2\pi r^2$) and "paint the paperweight" (solid, TSA $3\pi r^2$) both say "paint" and both are hemispheres. The deciding column is the third: which surfaces exist. That is why the formula comes last.

✎ EXPLAIN IT IN YOUR OWN WORDS

Pick any two rows that share a solid. In your own words, explain why their formulas differ.

Combined solids: volumes add, surfaces do not

Learning objective: Split an object into two solids, then treat its volume and its surface differently.



◆ UNDERSTAND

The critical rule. Volume: if the two parts do not overlap, volume of object = volume of part 1 + volume of part 2. (If a part is *removed*, subtract it.)
Surface: count only the surfaces on the **outside** of the finished object. A face that is glued to the other part is inside the object and is not counted.

WORKED EXAMPLE **Three different operations, one object (cylinder r = 7 cm, h = 10 cm, hemisphere on top)**

Operation	Working	Result	Is it the answer?
Add the volumes	1540 + 718.67	2258.67 cm³	✓ volume of the object
Add the two TSAs	748 + 462	1210 cm²	✗ counts the joint twice
Add exposed surfaces	308 + 440 + 154	902 cm²	✓ surface of the object

Check: 1210 − 902 = 308 = 2 × 154, exactly two copies of the hidden circle.

The nine-step method for any combined solid

1	Observe the whole object.	4	List given dimensions (and find hidden ones: hemisphere height = r).	7	Choose a formula for each part.
2	Name the two component solids.	5	Identify the quantity asked.	8	Add or subtract logically.
3	Sketch the parts separately.	6	Volume? Add parts. Surface? Draw a surface map.	9	Check units and size.

⚠ COMMON TRAP

"Volumes add, so surfaces add." Volumes are the space inside; joining keeps all of it. Surfaces are the skin outside; joining buries part of it.

Mistake clinic (1 of 3): measurements

MISTAKE 1 · Diameter used as radius

✗ STUDENT'S WORK	*Volume of a sphere of diameter 14 cm.* "⁴⁄₃ × 22/7 × 14³ = 11 498.67 cm³"
⚠ WHY IT IS WRONG	The formula needs r = 7 cm. Cubing the diameter makes the volume 8 times too big.
✓ CORRECTED	r = 7 cm; V = ⁴⁄₃ × 22/7 × 343 = 1437.33 cm³.
➤ QUICK RULE	Write "r = d ÷ 2 = ..." first.
✎ FIX IT E1	Find the surface area of a sphere of diameter 21 cm.

MISTAKE 2 · CSA given when TSA is needed

✗ STUDENT'S WORK	*Tin to make a closed can, r = 7 cm, h = 10 cm.* "2πrh = 440 cm²"
⚠ WHY IT IS WRONG	A closed can also has a top and a bottom, made of tin too.
✓ CORRECTED	TSA = 2πr(r + h) = 44 × 17 = 748 cm².
➤ QUICK RULE	"Make a closed..." → every face.
✎ FIX IT E2	Find the TSA of a closed cylinder with r = 14 cm and h = 6 cm.

MISTAKE 3 · Slant height in the cone's volume

✗ STUDENT'S WORK	*Cone r = 6 cm, l = 10 cm; volume.* "⅓ × π × 36 × 10 = 120π cm³"
⚠ WHY IT IS WRONG	Volume needs the perpendicular height h, not l.
✓ CORRECTED	h = √(100 − 36) = 8 cm; V = ⅓π × 36 × 8 = 96π cm³.
➤ QUICK RULE	Volume → h. Surface → l.
✎ FIX IT E3	A cone has r = 9 cm and l = 15 cm. Find its volume in terms of π.

MISTAKE 4 · Perpendicular height in the cone's CSA

✗ STUDENT'S WORK	*Cone r = 3 cm, h = 4 cm; CSA.* "π × 3 × 4 = 12π cm²"
⚠ WHY IT IS WRONG	The curved surface lies along the slope, so it needs l = √(9 + 16) = 5 cm.
✓ CORRECTED	CSA = π × 3 × 5 = 15π cm².
➤ QUICK RULE	Before πrl, check you have l.
✎ FIX IT E4	Find the CSA of a cone with r = 5 cm and h = 12 cm, in terms of π.

Practice 1: recognise

Learning objective: Recognise solids, quantities, measurements and units before calculating anything.

1 Recognise | 2 Apply | 3 Connect | 4 Board-style *Mostly no calculation. Answers: Section J.*

P1 The unit of the curved surface area of a cone, when lengths are in cm, is:

(a) cm (b) cm² (c) cm³ (d) cm⁴

Purpose: Units match quantities. **easy ★☆☆**

Working space (use separate paper if needed)

P2 The volume of a hemisphere of radius r is:

(a) $2\pi r^2$ (b) $3\pi r^2$ (c) $\frac{2}{3}\pi r^3$ (d) $\frac{4}{3}\pi r^3$

Purpose: Tell surface formulas from volume formulas. **easy ★☆☆**

Working space (use separate paper if needed)

P3 Name the two solids that model each object: (a) a circus tent (b) a medicine capsule (c) an ice-cream cone with a scoop on top (d) a sharpened pencil (e) a test tube.

Purpose: Decompose real objects. **easy ★☆☆**

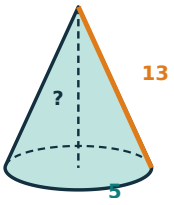
Working space (use separate paper if needed)

P4 CSA, TSA, volume or another surface? (a) paint the outside of a closed water tank (b) the water the tank can hold (c) a label round a can (d) canvas for a tent with no floor (e) silver to coat a ball.

Purpose: Decide what is asked. **moderate ★★☆**

Working space (use separate paper if needed)

P5 The cone in the figure has radius 5 cm and slant height 13 cm. (a) Find the perpendicular height. (b) Which height goes into the CSA formula, and which into the volume formula?



Purpose: Two heights of a cone. **moderate ★★☆**

Working space (use separate paper if needed)

P6 Two cubes of edge 3 cm are joined face to face. The surface area of the new solid is:

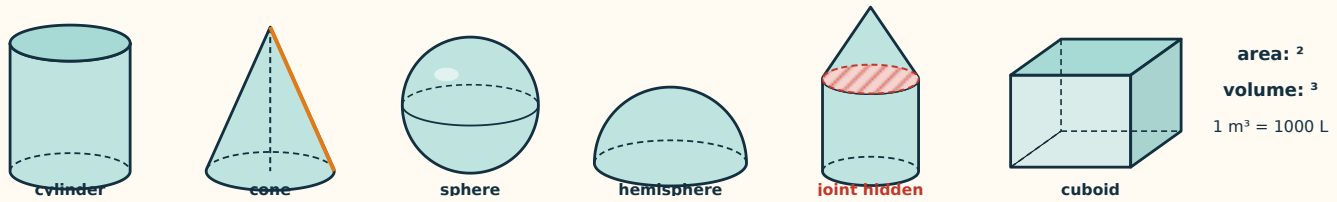
(a) 108 cm² (b) 90 cm² (c) 72 cm² (d) 54 cm²

Purpose: Hidden faces. **moderate ★★☆**

Working space (use separate paper if needed)

Revision sheet

Learning objective: Recall every prescribed formula with its condition, on one page.



Solid	Curved / lateral	Total surface	Volume	Remember
Cuboid l, b, h	2(l + b)h	2(lb + bh + hl)	lbh	Open box: lb + 2(l + b)h
Cube a	$4a^2$	$6a^2$	a^3	Joined cubes hide faces
Cylinder r, h	2πrh	2πr(r + h)	$\pi r^2 h$	Open top: 2πrh + πr²; pipe: 2πrh
Cone r, h, l	πrl	πr(l + r)	$\frac{1}{3}\pi r^2 h$	$l^2 = r^2 + h^2$; area → l, volume → h
Sphere r		$4\pi r^2$	$\frac{4}{3}\pi r^3$	No flat face
Hemisphere r	$2\pi r^2$	$3\pi r^2$ (solid)	$\frac{2}{3}\pi r^3$	Bowl → $2\pi r^2$; height = r

⚖ KNOWING ≠ CHOOSING

Knowing says: the formula table above

1 · WHICH SOLID?	Name it (or the two solids).
2 · WHAT IS ASKED?	Covering → area. Filling, holding, metal → volume. Rim, ribbon → length.
3 · WHICH SURFACES?	Open? Joined? Removed part? Build the surface from the faces that exist.
4 · WHICH MEASUREMENTS?	r (not d); for a cone, l or h as the formula needs; one unit for everything.

SO CHOOSE **Only now write the formula, then substitute.**

◐ INSIDE OR OUTSIDE?

Combined solids. Volume: add the parts (subtract any removed part). Surface: only outside faces. Joined circle → not counted. Smaller solid on a bigger face → $-\pi r^2$ (covered) + its exposed surface. Scooped out → new inner surface added.

◆ RULES

Recasting ENRICHMENT: volume before = volume after (× number of pieces), assuming no loss. Never equate surface areas.
Units: 1 m = 100 cm; 1 m^2 = 10 000 cm^2; 1 m^3 = 1 000 000 cm^3 = 1000 L; 1 L = 1000 cm^3.

Object in the question	Model	Usually asked	Watch for
tin, drum, pillar, pipe, well	cylinder	TSA / CSA / volume	open ends; well → volume of earth
tent, rocket, pencil, silo	cylinder + cone	canvas / paint; air or grain	no floor; find l
capsule, test tube, gulab jamun	cylinder + hemisphere(s)	surface; volume	cylinder length = total − r (or − 2r)
toy top, ice-cream with a scoop; bowl, dome	cone + hemisphere; open hemisphere	paint; volume; capacity	cone height = total − r; a bowl has no flat face

⚠ COMMON TRAP

Top mistakes: d used as r · l in a cone's volume · h in πrl · bowl given $3\pi r^2$ · closed TSA for open objects · joints counted · TSAs added · units mixed · ² for volume · wrong π · early rounding · formula with no reason.

MATHS WITHOUT GUESSWORK

Statistics

Understand the Data.
Choose the Method.
Solve Independently.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27

12 35 27 8
41 19 23 30
16 44 28 33
5 21 38 25
…
raw data

→

Marks	Students f_i	Cumulative frequency (cf)
0 – 10	5	5
10 – 20	8	13
20 – 30	15	28
30 – 40	12	40
40 – 50	6	46
50 – 60	4	50
Total	**$\Sigma f_i = 50$**	

→ **mean** 28.6 **median** 28 **mode** 27

Recognise the method first. The calculation comes second.

Core topics: mean (direct, assumed mean, step-deviation) · median · mode of grouped data

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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From raw data to a frequency table

PREREQUISITE

Learning objective: Turn a list of observations into a grouped frequency distribution, and read it correctly.

◆ UNDERSTAND

Statistics is about collecting, organising and interpreting data. **Raw data** is the list exactly as collected. A **frequency distribution** organises it: how many observations (the **frequency**) fall in each group (the **class interval**).

Raw data: marks of 30 students (out of 50).

12, 35, 27, 8, 41, 19, 23, 30, 16, 44, 28, 33, 5, 21, 38, 25, 14, 47, 29, 36, 22, 18, 31, 26, 9, 42, 24, 34, 17, 27

Marks	Tally	Frequency
0 − 10	|||	3
10 − 20	||||/ |	6
20 − 30	||||/ ||||/	10
30 − 40	||||/ ||	7
40 − 50	||||	4
Total		**30**

★ SPOT THE CLUE

Which class does 30 go in? In NCERT tables such as 20 − 30, 30 − 40, a value equal to an upper limit belongs to the **next** class. So 30 is counted in 30 − 40, not 20 − 30.

? WHY THIS STEP?

Why group the data? 30 separate marks are hard to read. Five classes show the shape at once: most students scored between 20 and 40. The price is that we no longer know each exact mark, so grouped-data methods *assume* the values in a class are centred at its middle (its class mark).

✓ CHECK BEFORE YOU CALCULATE

Check the total. The frequencies must add up to the number of observations: 3 + 6 + 10 + 7 + 4 = 30 ✓. If they do not, a value was missed or counted twice.

✓ CHECK YOURSELF

C1. Using the raw data above, regroup the marks into classes 0 − 25 and 25 − 50. Write the two frequencies and check that they add up to 30.

Concept 1: the direct method, step by step

CORE

Learning objective: Build the table, substitute, and check that the mean is sensible.

WORKED EXAMPLE **Find the mean marks of 40 students (table on page 12).**

1	Class marks: (0 + 10) ÷ 2 = 5, (10 + 20) ÷ 2 = 15, (20 + 30) ÷ 2 = 25, (30 + 40) ÷ 2 = 35, (40 + 50) ÷ 2 = 45	Represent each class by its centre.
2	Products: 3 × 5 = 15, 7 × 15 = 105, 12 × 25 = 300, 10 × 35 = 350, 8 × 45 = 360	Estimated total marks of each class.
3	$\Sigma f_i = 40$; $\Sigma f_i x_i = 15 + 105 + 300 + 350 + 360 = 1130$	Total observations; total marks.
4	$\bar{x} = \Sigma f_i x_i \div \Sigma f_i = 1130 \div 40 =$ **28.25**	Share the total equally.

≈ ESTIMATE AND CHECK

Is 28.25 sensible? It must lie between the smallest and largest class marks (5 and 45), and near the tallest classes (20 − 40 hold 22 of 40 students). ✓

✓ CHECK BEFORE YOU CALCULATE

Before dividing: (1) Σf_i equals the number of observations in the question. (2) Each class mark is exactly halfway. (3) Each product uses the frequency **from the same row**.

→ PREDICT THE NEXT STEP

You have finished the x_i column. What comes next, and what will you add up at the end? (The $f_i x_i$ column; then Σf_i and $\Sigma f_i x_i$.)

Daily wage (₹ in tens)	Workers f_i
10 − 20	4
20 − 30	6
30 − 40	10
40 − 50	12
50 − 60	8
Total	**$\Sigma f_i = 40$**

✎ YOUR TURN

T1. The table shows the daily wages (in tens of rupees) of 40 workers. Find the mean daily wage by the direct method. Show the x_i and $f_i x_i$ columns.

Concept 4: the median of grouped data

CORE

Learning objective: Locate the median class with cumulative frequency, and understand every symbol of the median formula.

Hold this question: *The median is the middle observation. In a grouped table, how do I know which class it is in?*

◆ UNDERSTAND

The median is the value with **half the observations below it**: the (n/2)-th observation. Cumulative frequency counts how many observations lie below each upper limit, so the first class whose cf passes n/2 must contain the median. This is the **median class**. NCERT: the class whose cumulative frequency is **greater than (and nearest to) n/2**.

Marks	Students f_i	Cumulative frequency (cf)	
0 – 10	4	4	
10 – 20	6	10	
20 – 30	10	20	◀ cf
30 – 40	**16**	**36**	◀ median class
40 – 50	8	44	
50 – 60	4	48	
60 – 70	2	50	
Total	**$\Sigma f_i = 50$**		

▶ WHICH CLASS SHOULD I CHOOSE?

n = 50, n/2 = 25. Running down the cf column: 4, 10, 20, 36. The first cf greater than 25 is **36**, so the median class is **30 – 40**. The class above it has cf = 20: that many observations come *before* the median class.

➤ THE MEDIAN FORMULA

$$\text{Median} = l + ((n/2 - cf) \div f) \times h$$

Symbol	Meaning	Where to find it	Here
l	lower limit of the median class	the orange row	**30**
n	total frequency	total of f column	**50**
cf	cumulative frequency of the class **before** the median class	the pale orange row	**20**
f	frequency of the median class	the orange row	**16**
h	class size	upper − lower limit	**10**

? WHY THIS STEP?

Why "n/2 − cf"? 20 observations are below the median class. We need 25 below the median, so we must go a further 5 observations into the class. The class holds f = 16 observations spread over h = 10 marks, so we move (5 ÷ 16) of the way across it.

Doubts 1 and 2: which measure? frequency or cf?

DOUBT 1 — *"How do I know whether to find the mean, the median or the mode?"*

✗ WHY THIS CONFUSION HAPPENS

All three are "averages" in everyday speech, and questions sometimes just say "find the average value".

◆ UNDERSTAND

Mean = the total shared equally. Median = the middle observation. Mode = the most common value. Decide from what the answer will be *used* for.

★ SPOT THE CLUE

"average, mean, per day" → mean · "middle, half, 50% below" → median · "most, most popular, modal, typical size sold" → mode.

➤ CHOOSE THE METHOD

Mean: $\Sigma f_i x_i \div \Sigma f_i$ (or a shortcut). Median: cf + formula. Mode: modal class + formula.

✎ SOLVED EXAMPLE

"A shop wants to stock the quantity it sells **most often**." → mode. Modal class 100 − 150: Mode = 100 + (22 − 14) ÷ (2 × 22 − 14 − 10) × 50 = 100 + 8/20 × 50 = **120**.

Units sold	Days f_i
0 − 50	8
50 − 100	14
100 − 150	22
150 − 200	10
200 − 250	6
Total	**$\Sigma f_i = 60$**

? WHY THIS STEP?

The shop does not care about total sales here, only about the most common day, so the mode answers its question.

⚠ COMMON TRAP

Finding the mean (118.33) because it is the "real average". It answers a different question.

✎ YOUR TURN DB1

Members' ages: classes 20 − 30, …, 60 − 70 with frequencies 6, 9, 15, 12, 8. (a) Which measure tells you "half the members are younger than ___"? (b) Find it.

✓ CHECK YOURSELF

Half below → median. Your answer must lie in the class where cf first passes n/2.

DOUBT 2 — *"What is the difference between frequency and cumulative frequency?"*

✗ WHY THIS CONFUSION HAPPENS

Both are counts in columns next to each other, and both are called "frequency".

◆ UNDERSTAND

Frequency counts the observations **in** one class. Cumulative frequency counts all observations **up to the end of** that class.

★ SPOT THE CLUE

"How many in 10 − 15?" → frequency. "How many less than 15?" → cumulative frequency.

➤ CHOOSE THE METHOD

Build cf by adding each frequency to the previous cf. The last cf must equal n.

✎ SOLVED EXAMPLE

Hours online: classes 0 − 5, …, 20 − 25 with frequencies 7, 11, 15, 9, 8. cf: 7, 18, 33, 42, 50. Students online less than 15 hours = **33**; in 10 − 15 alone = **15**.

? WHY THIS STEP?

The running total answers every "less than" question at once, which is exactly what the median needs.

⚠ COMMON TRAP

Writing the frequency again in the cf column, or forgetting the first row (cf of the first class = its own frequency).

✎ YOUR TURN DB2

Weights: classes 30 − 40, …, 70 − 80 with frequencies 5, 12, 18, 9, 6. Write the cf column. How many people weigh less than 60 kg? How many weigh 50 kg or more?

✓ CHECK YOURSELF

The last cf must be 50. "50 kg or more" = n − (cf less than 50).

Question clue → measure → table → formula → first step

Learning objective: Translate the words of a question into a measure, the columns to build, and a first step.

Question clue	Required measure	Table preparation	Formula	First step
"Find the mean / average ..."	**mean**	x_i, f_ix_i	$\Sigma f_i x_i \div \Sigma f_i$	class marks
"... by the assumed mean method"	**mean**	x_i, d_i, f_id_i	$a + \Sigma f_i d_i \div \Sigma f_i$	choose a central x_i
"... by the step-deviation method"	**mean**	x_i, u_i, f_iu_i	$a + h \times \Sigma f_i u_i \div \Sigma f_i$	check h divides every d_i
"The mean is 54; find the missing frequency"	**mean (reversed)**	x_i, f_ix_i with p	**equation in p**	write Σf and Σfx with p
"Find the median / middle value"	**median**	cf	$l + ((n/2 - cf) \div f) \times h$	n/2
"Half the ... are below what value?"	**median**	cf	**median formula**	n/2
"Less than 10: 3, less than 20: 11, ..."	**median (usually)**	recover f, then cf	**median formula**	subtract consecutive cf
"The median is 28 and n = 60; find x and y"	**median (reversed)**	cf with x, y	**median formula + Σf = n**	locate median class
"Find the mode / modal ..."	**mode**	none extra	$l + ((f_1 - f_0) \div (2f_1 - f_0 - f_2)) \times h$	find the highest f
"Most popular / most common ..."	**mode**	none extra	**mode formula**	find the highest f
"Classes 10 − 19, 20 − 29, ..."	**any**	class boundaries	**same formulas, l from boundaries**	subtract/add 0.5
"Find the mean, median and mode and compare"	**all three**	x_i, f_ix_i and cf	**three formulas**	n, then build all columns once

➤ WHY THIS FORMULA?

Read the table from left to right, never right to left. The formula is the result of the first three columns. Two rows can share a formula (median) and still need different first steps, because the table arrives in a different form.

✓ CHECK BEFORE YOU CALCULATE

Before calculating anything, write one line: "Measure: ___ · Columns: ___ · Method: ___ because ___." It takes 15 seconds and prevents most wrong-method answers.

Mistake clinic (1 of 3): reading the table

MISTAKE 1 · Class mark taken as the upper limit

✗ STUDENT'S WORK	*Mean of classes 10 − 20, ..., 40 − 50; frequencies 3, 5, 8, 4.* "x_i = 20, 30, 40, 50; $\bar{x}$ = 36.5"
⚠ WHY IT IS WRONG	The class mark is the midpoint. Upper limits push every value up by half a class.
✓ CORRECTED	x_i = 15, 25, 35, 45; $\bar{x}$ = 630 ÷ 20 = 31.5.
➤ QUICK RULE	x_i = (lower + upper) ÷ 2. Check: it is exactly halfway.
✎ FIX IT E1	Find the mean of classes 20 − 40, ..., 80 − 100; frequencies 4, 6, 7, 3.

MISTAKE 2 · Frequency used as the class mark

✗ STUDENT'S WORK	*Mean of a table.* "$\bar{x} = \Sigma f_i \times f_i \div \Sigma f_i$"
⚠ WHY IT IS WRONG	Frequencies say how many; class marks say how big. Multiplying frequency by itself has no meaning.
✓ CORRECTED	Build a separate x_i column from the class limits; then use $f_i x_i$.
➤ QUICK RULE	Headings: "how many" column (f) and "how big" column (x).
✎ FIX IT E2	Write the class marks of 0 − 8, 8 − 16, 16 − 24. What is the class width?

MISTAKE 3 · Dividing by the number of classes

✗ STUDENT'S WORK	*Mean of classes 0 − 10, ..., 40 − 50; frequencies 6, 9, 14, 8, 3.* "$\Sigma f_i x_i$ = 930; $\bar{x}$ = 930 ÷ 5 = 186"
⚠ WHY IT IS WRONG	The total must be shared among n = 40 observations, not 5 classes. 186 is outside the data.
✓ CORRECTED	$\bar{x}$ = 930 ÷ 40 = 23.25.
➤ QUICK RULE	Divide by Σf_i. Check the mean lies within the class marks.
✎ FIX IT E3	Find the mean of classes 0 − 5, ..., 20 − 25; frequencies 3, 7, 11, 6, 3.

MISTAKE 4 · Median class taken as the tallest class

✗ STUDENT'S WORK	*Median of classes 0 − 10, ..., 50 − 60; frequencies 14, 6, 9, 11, 7, 3.* "Median class = 0 − 10 (highest f)... median = 17.86"
⚠ WHY IT IS WRONG	n/2 = 25; cf: 14, 20, 29, 40, 47, 50. The first cf above 25 is in 20 − 30.
✓ CORRECTED	n = 50, n/2 = 25; median class 20 − 30; l = 20 cf = 20 f = 9 h = 10 Median = 20 + (25 − 20) ÷ 9 × 10 = 20 + 5.5556 = **25.56**
➤ QUICK RULE	Median class: first cf greater than n/2. Never by height.
✎ FIX IT E4	Find the median of classes 10 − 20, ..., 50 − 60; frequencies 14, 9, 7, 6, 4.

Practice 1: recognise

Learning objective: Read tables, recognise the measure and locate the right classes before calculating.

1 Recognise | 2 Apply | 3 Connect | 4 Board-style — *Little or no calculation. Answers: Section H.*

P1 The class mark of the class $15 - 25$ is:

(a) 15 (b) 20 (c) 25 (d) 10

Purpose: Class marks. **easy ★☆☆**

P2 A cumulative frequency column is needed to find the:

(a) mean (b) median (c) mode (d) class width

Purpose: Columns follow the measure. **easy ★☆☆**

P3 For the table, find (a) the total frequency (b) the class width (c) all class marks.

Purpose: Read a table. **easy ★☆☆**

Class interval	Frequency f_i
$0 - 10$	7
$10 - 20$	12
$20 - 30$	18
$30 - 40$	9
$40 - 50$	4
Total	**$\Sigma f_i = 50$**

Working space (use separate paper if needed)

P4 Complete the cumulative frequency column for this table. What must the last cf equal?

Purpose: Build cf. **easy ★☆☆**

Marks	Frequency f_i
$10 - 20$	5
$20 - 30$	8
$30 - 40$	12
$40 - 50$	9
$50 - 60$	6
Total	**$\Sigma f_i = 40$**

Working space (use separate paper if needed)

Revision sheet

Learning objective: Recall every prescribed method, its symbols and its first step, on one page.

Term	Meaning	Term	Meaning
class mark x_i	(lower + upper limit) ÷ 2	class size h	upper − lower limit
$n = \Sigma f_i$	total frequency	cumulative frequency	running total; last cf = n
median class	first class with cf > n/2	modal class	class with the highest frequency

Measure / method	Formula	Columns	First step
Mean, direct	$\bar{x} = \Sigma f_i x_i \div \Sigma f_i$	x_i, $f_i x_i$	class marks
Mean, assumed mean	$\bar{x} = a + \Sigma f_i d_i \div \Sigma f_i,\ d_i = x_i - a$	x_i, d_i, $f_i d_i$	choose a central x_i as a
Mean, step-deviation	$\bar{x} = a + h \times \Sigma f_i u_i \div \Sigma f_i,\ u_i = d_i \div h$	x_i, u_i, $f_i u_i$	check h divides every d_i
Median	$l + ((n/2 - cf) \div f) \times h$	cf	n/2, then the median class
Mode	$l + ((f_1 - f_0) \div (2f_1 - f_0 - f_2)) \times h$	none	highest f, then f_0 above, f_2 below

◆ RECOGNISE THE METHOD

Rushing says: "Use the formula I remember best."

1 · WHAT IS ASKED?	Average · middle · most common?
2 · WHICH MEASURE?	Mean · median · mode.
3 · WHICH COLUMNS?	x_i (and d_i/u_i) · cf · nothing extra.
4 · WHICH METHOD, WHY?	Direct if numbers are small; shortcut if class marks are large; median/mode by their formulas.

SO USE **Only then: build columns, list values, substitute.**

◆ CONVENTIONS

Table conventions. A value equal to an upper limit belongs to the next class. Inclusive classes (10 − 19, 20 − 29): use boundaries (9.5 − 19.5, ...) for l and h in median and mode. Cumulative tables: recover f by subtracting. Bimodal tables are avoided in the syllabus. 3 Median ≈ Mode + 2 Mean is an NCERT remark (empirical), not a required formula.

⚠ COMMON TRAP

Top mistakes: upper limit as class mark · dividing by number of classes · median class = tallest class · cf of the median class itself · l = upper limit · f_0/f_2 swapped · forgetting × h · lost minus signs · skipped cf row · early rounding · inclusive classes not converted.

✓ CHECK BEFORE YOU CALCULATE

Before every final answer: $\Sigma f_i = n$? last cf = n? mean between the smallest and largest class marks? median inside the median class? mode inside the modal class? two mean methods agree?

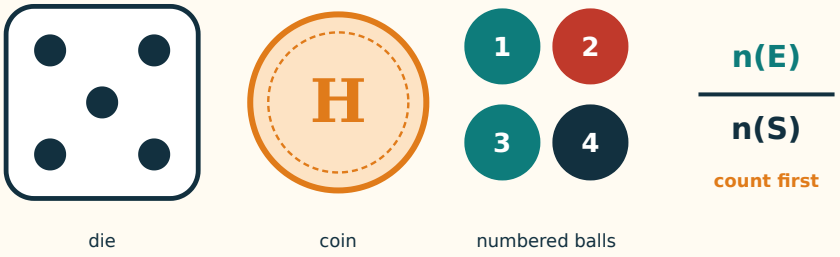
MATHS WITHOUT GUESSWORK

Probability

Understand the Outcomes.
Count Carefully.
Solve Independently.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27



die coin numbered balls

Count the right outcomes first. The formula is the easy part.

Core topics: classical definition of probability · simple problems on finding the probability of an event

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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Events and favourable outcomes

PREREQUISITE

Learning objective: Turn a sentence into an event and pick out exactly the outcomes that make it happen.

◆ MEANING	An **event** E is a description of what we are looking for. The outcomes that make the event happen are its **favourable outcomes**; their number is **n(E)**.
★ EVERYDAY EXAMPLE	"Our team wins the toss" is an event; it happens for one outcome (our call comes up).
➤ IN MATHEMATICS	Die, event "an even number": 1 **2** 3 **4** 5 **6** favourable 2, 4, 6, so n(E) = 3.
⚠ COMMON CONFUSION	"Favourable" does **not** mean "good for me". If the event is "getting a 1", then 1 is favourable even if 1 loses the game.
✎ QUICK PRACTICE	Die, event "a number less than 3": write the favourable outcomes.

⇄ SAME EXPERIMENT, DIFFERENT EVENT

One die, S = {1, 2, 3, 4, 5, 6}. The total stays 6; only the favourable outcomes change.

Event	Outcomes (teal = favourable)	n(E)	P(E)
an even number	1 **2** 3 **4** 5 **6**	3	$\frac{1}{2}$
the number 4	1 2 3 **4** 5 6	1	$\frac{1}{6}$
a prime number	1 **2** **3** 4 **5** 6	3	$\frac{1}{2}$
the number 8	1 2 3 4 5 6	0	0

◆ UNDERSTAND

An event with exactly **one** favourable outcome is an **elementary event** (e.g. "getting 4"). A die has six elementary events, each with probability 1/6, and they add up to 1.

✔ CHECK YOURSELF

C2. A die is thrown. List the favourable outcomes and n(E) for: (a) a number less than 3; (b) a multiple of 3; (c) a number greater than 6.

The formula, and the eight steps that come before it

CORE

Learning objective: Name the numerator and denominator of P(E) and follow a fixed order of steps.

➤ FORMULA WITHOUT GUESSWORK

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of outcomes favourable to E}}{\text{total number of equally likely outcomes}}$$

P(E): probability of event E · **n(E)**: how many outcomes make E happen (numerator) · **n(S)**: how many equally likely outcomes the experiment has (denominator)

1	Read the question carefully.	Underline the event words.
2	Identify the experiment and the event.	What is done once? What are we looking for?
3	Determine all possible outcomes.	List, tree or grid.
4	Identify which outcomes satisfy the event.	Test each outcome against the event.
5	Count the favourable outcomes: n(E).	Count once each.
6	Count the total equally likely outcomes: n(S).	Check with 2^k, 6 × 6, 52, etc.
7	Apply the formula and simplify.	Numerator = n(E), denominator = n(S).
8	Check that the answer lies between 0 and 1.	And that n(E) + n(not E) = n(S).

List, or count systematically?

Situation	Best tool	Example
Few outcomes (up to about 12)	write every outcome	die; two or three coins; coin + die
Two objects, 36 outcomes	6 × 6 grid	two dice; a die thrown twice
Many identical-type outcomes	count with a rule	numbers 1 to 90: two-digit numbers are 10 to 90, i.e. 90 − 9 = 81
Objects in groups	add the group sizes	bag with 4 red, 6 blue, 2 green: n(S) = 12
Cards	use the deck structure	4 suits × 13 ranks; 12 face cards

◆ UNDERSTAND

Four distinctions this chapter keeps testing: counting **objects** vs counting **colour names**; counting **favourable** vs **all** outcomes; counting each outcome **once** vs twice; a **complete** sample space vs one with a gap. Each worked example below highlights one of them.

Worked example 8: two dice and the 6 × 6 grid

CORE

Learning objective: Use the grid to count all 36 ordered outcomes and select the favourable ones.

WORKED EXAMPLE 8 **A blue die and a grey die are thrown together. Find P(sum is 8), P(sum is 13), P(sum is 12 or less), P(a doublet).**

WHAT IS ASKED?	Four probabilities, all about the pair of numbers.
WHAT IS GIVEN?	Two fair dice that can be told apart.
HOW DO I IDENTIFY THE OUTCOMES?	(blue, grey) ordered pairs in a 6 × 6 grid.
WHY IS THIS METHOD SUITABLE?	Each of the 36 pairs is equally likely; the sums 2 to 12 are not.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	**(2,6)**
3	(3,1)	(3,2)	(3,3)	(3,4)	**(3,5)**	(3,6)
4	(4,1)	(4,2)	(4,3)	**(4,4)**	(4,5)	(4,6)
5	(5,1)	(5,2)	**(5,3)**	(5,4)	(5,5)	(5,6)
6	(6,1)	**(6,2)**	(6,3)	(6,4)	(6,5)	(6,6)

◆ **COUNT BEFORE YOU CALCULATE**

1 · THE EXPERIMENT	Two dice thrown together.
2 · ALL OUTCOMES · n(S)	the grid: 6 × 6 = 36 ordered pairs
3 · THE EVENT IN WORDS	the sum is 8
4 · FAVOURABLE · n(E)	(2, 6), (3, 5), (4, 4), (5, 3), (6, 2) → n(E) = 5

FORMULA WITHOUT GUESSWORK $P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$

Event	Favourable outcomes	n(E)	P(E)
the sum is 13	none	0	0/36 = 0
the sum is 12 or less	(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), …, (6, 6)	36	36/36 = 1
a doublet (same number on both)	(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)	6	6/36 = 1/6

⚠ **COMMON TRAP**

Counting (2, 6) and (6, 2) as one outcome. They are different cells of the grid (blue 2, grey 6 vs blue 6, grey 2).

Final answer (board format): P(sum 8) = 5/36; P(sum 13) = 0 (impossible); P(sum ≤ 12) = 1 (certain); P(doublet) = 1/6.

✎ **YOUR TURN**

T7. Two dice are thrown. Find P(the sum is 10) and P(the product is 6).

✓ **WHY IS THIS FAVOURABLE?**

A doublet means the same number on both dice: the diagonal (1, 1), (2, 2), …, (6, 6): 6 cells. The largest possible sum is 6 + 6 = 12, so a sum of 13 has no cells (impossible) and "12 or less" is every cell (certain).

UNIT 1 · UNDERSTANDING THE QUESTION

Doubts 1 and 2: what am I supposed to count?

DOUBT 1 — *"How do I identify the event in a long question?"*

✗ WHY THIS CONFUSION HAPPENS	Questions wrap the event in a story; students count what is mentioned last or loudest.
◆ UNDERSTAND THE IDEA	The event is the condition the outcome must satisfy. Strip the story and write "E: ..." in a few words.
★ SPOT THE CLUE	Words after "probability that..." or "probability of getting...". Joining words: "both ... and", "or", "not".
➤ CHOOSE THE METHOD	Write the event in words, then test each outcome against it.
✎ SOLVED EXAMPLE	A number is picked at random from 1 to 30. n(S) = 30. Event: a multiple of both 2 and 3. Favourable: 6, 12, 18, 24, 30; n(E) = 5. P = 5/30 = **1/6**. "Both 2 and 3" means divisible by 6.
? WHY THIS STEP?	"Multiple of both 2 and 3" needs both tests passed; 4 passes one test only, so it is not favourable.
⚠ COMMON TRAP	Counting multiples of 2 and multiples of 3 separately (15 + 10) and adding: that is a different event, and it double-counts.
✎ YOUR TURN DB1	A number is chosen at random from 1 to 25. Find P(it is a multiple of both 3 and 4).
✓ CHECK YOURSELF	a multiple of both 3 and 4: **2/25** (2 out of 25) — only 12 and 24.

DOUBT 2 — *"Does "favourable" mean the result I want or like?"*

✗ WHY THIS CONFUSION HAPPENS	In everyday English, favourable means good for me.
◆ UNDERSTAND THE IDEA	In probability, favourable means **satisfies the event**, even if that result is bad for you.
★ SPOT THE CLUE	Read the event, not the story: "the probability that she does **not** win" makes losing outcomes favourable.
➤ CHOOSE THE METHOD	Favourable = outcomes making the stated event true.
✎ SOLVED EXAMPLE	A die is thrown once. n(S) = 6. Event: not getting a 6. Favourable: 1, 2, 3, 4, 5; n(E) = 5. P = 5/6. Rahul wants a 6, but the event asked is "not 6".
? WHY THIS STEP?	We count outcomes that make the *event* happen. Here that is 1, 2, 3, 4, 5.
⚠ COMMON TRAP	Answering 1/6 because "6 is the good outcome for Rahul".
✎ YOUR TURN DB2	Asha wins only if a dial numbered 1 to 8 stops at 8. Find P(Asha does not win).
✓ CHECK YOURSELF	not 8 (Asha does not win): **7/8** (7 out of 8)

If the question says… then identify… then do…

Learning objective: Translate the words of the question into a counting plan.

If the question says or shows…	What should I identify?	What should I do next?
A coin is tossed (once, twice, three times)	H/T for each toss, in order	List in order: 2, 4 or 8 outcomes
A die is rolled once	the six faces	Select the faces satisfying the condition
Two dice / a die thrown twice	ordered pairs (first, second)	Use the 6 × 6 grid; 36 outcomes
A coin and a die together	pairs (coin, die)	List 2 × 6 = 12 pairs
An object is selected from a bag or box	each object (not each colour)	Add group sizes for n(S); count qualifying objects
A card is drawn from a deck	suit, colour, rank; 52 cards	Count by structure: 13 per suit, 4 per rank, 12 face cards
Cards are removed first	the cards that remain	Rebuild n(S) and n(E) from what is left
A number condition (prime, square, multiple)	exactly which numbers qualify	List or count by rule; check boundaries
"Not" a particular event	outcomes excluded by the condition	n(S) − n(E), or 1 − P(E)
"At least one" / "exactly one"	how many successes are allowed	"at least one" = all − none; "exactly" = only that number
"Or" / "neither … nor"	the overlap of the two groups	n(A) + n(B) − n(both); neither = n(S) − that
A second draw without replacement	what was removed	Update both counts before the second probability
A probability or a ratio is given; find a number	the unknown count	Set n(E)/n(S) equal to the given value and solve
A long, complicated condition	the event in simple words	Break it into tests joined by and / or / not

Before you calculate, ask yourself

1. Have I understood the event?

2. Have I counted all possible outcomes?

3. Are the outcomes equally likely?

4. Have I identified only the favourable outcomes?

5. Have I avoided double-counting?

6. Is n(E) in the numerator and n(S) in the denominator?

7. Is my final probability between 0 and 1?

8. Is it in simplest form?

Mistake clinic (1 of 3): counting the outcomes

MISTAKE 1 · Favourable and total swapped	
✗ INCORRECT APPROACH	*Bag: 7 green, 3 yellow. P(green)?* "P = 10/7"
⚠ WHY IT FAILS	The total (10) went on top. The answer is above 1, which is impossible.
✓ CORRECT APPROACH	n(E) = 7 green balls; n(S) = 10 balls. P(green) = 7/10.
➤ EXAM-READY LINE	P(green) = n(E)/n(S) = 7/10.
✎ FIX IT E1	Same bag. Find P(yellow).

MISTAKE 2 · Forgetting outcomes	
✗ INCORRECT APPROACH	*Two coins. P(two heads)?* "S = {HH, HT, TT}, P = 1/3"
⚠ WHY IT FAILS	TH is missing; 2 × 2 = 4 outcomes. The three listed are not equally likely.
✓ CORRECT APPROACH	S = {HH, HT, TH, TT}; P(HH) = 1/4.
➤ EXAM-READY LINE	n(S) = 4, n(E) = 1, P = 1/4.
✎ FIX IT E2	Two coins. Find P(two tails).

MISTAKE 3 · Counting an outcome twice	
✗ INCORRECT APPROACH	*P(a heart or a king)?* "13 + 4 = 17, P = 17/52"
⚠ WHY IT FAILS	K♥ is a heart and a king: it was counted in both groups.
✓ CORRECT APPROACH	13 + 4 − 1 = 16; P = 16/52 = 4/13.
➤ EXAM-READY LINE	n(E) = 13 + 4 − 1 = 16 (K♥ counted once); P = 4/13.
✎ FIX IT E3	Find P(a spade or an ace).

MISTAKE 4 · Treating different outcomes as identical	
✗ INCORRECT APPROACH	*Two dice. P(sum 7)?* "Sums are 2 to 12: 11 outcomes, P = 1/11"
⚠ WHY IT FAILS	The 11 sums are not equally likely. Only the 36 ordered pairs are.
✓ CORRECT APPROACH	Sum 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) → 6/36 = 1/6.
➤ EXAM-READY LINE	n(S) = 36 ordered pairs; n(E) = 6; P = 1/6.
✎ FIX IT E4	Two dice. Find P(sum is 6).

Practice level 1: recognise

Learning objective: Identify outcomes, events and totals before any calculation.

1 Recognise | 2 Apply | 3 Connect and reason | 4 Board-style — *Little or no calculation.*

All questions in this section are **original practice** written for this workbook in the CBSE style; none is presented as an official previous-year question.

P1 Two coins are tossed together. The number of possible outcomes is:

(a) 2 (b) 3 (c) 4 (d) 8

Purpose: Size of S. **easy** ★☆☆

P2 A spinner has four equal sectors coloured red, green, blue and yellow. Write the sample space and n(S).

Purpose: Write S. **easy** ★☆☆

Working space

P3 A die is thrown. List the outcomes favourable to "a number less than 5".

Purpose: Favourable list. **easy** ★☆☆

Working space

P4 Which of these cannot be the probability of an event?

(a) 0.01 (b) 101/100 (c) 0 (d) 33%

Purpose: Range. **easy** ★☆☆

P5 Impossible, certain, or neither? (a) getting 7 on a die (b) getting a number less than 7 on a die (c) drawing a red card from a pile containing only spades (d) getting a head when a coin is tossed.

Purpose: Event types. **easy** ★☆☆

Working space

P6 If P(E) = 0.08, find P(not E).

Purpose: Complement. **easy** ★☆☆

Working space

P7 Two dice are thrown. How many outcomes are there? How many are doublets?

Purpose: Grid counts. **easy** ★☆☆

Working space

P8 In a deck of 52 cards, how many are (a) face cards (b) red cards (c) aces (d) black face cards?

Purpose: Deck structure. **easy** ★☆☆

Working space

Quick revision (1 of 2): definitions, formula, checks

Learning objective: Revise the whole chapter in 5–10 minutes. Nothing new here.

Term	Meaning	Example (one die)
Experiment	an action with uncertain result	throwing a die once
Outcome	one possible result	4
Sample space S, n(S)	the complete list of outcomes; how many	{1, ..., 6}; 6
Event E, n(E)	a condition; outcomes satisfying it; how many	"even": 2, 4, 6; 3
Elementary event	an event with one outcome	"getting 4"
Impossible / certain	n(E) = 0, P = 0 / n(E) = n(S), P = 1	"8" / "less than 7"
Complement (not E)	all outcomes where E fails	"not even": 1, 3, 5

➤ FORMULA WITHOUT GUESSWORK

$$P(E) = \frac{n(E)}{n(S)} \qquad 0 \le P(E) \le 1 \qquad P(\text{not } E) = 1 - P(E)$$

Valid when all outcomes are **equally likely**. Example: die, "a number greater than 4": n(E) = 2 (5, 6), n(S) = 6, P = 2/6 = 1/3.

▦ VISUAL OUTCOME ORGANISER

Sample-space sizes: 1 coin 2 · 2 coins 4 · 3 coins 8 · 1 die 6 · 2 dice (or die twice) 36 · coin + die 12 · deck 52 (26 red, 26 black; 13 per suit; 4 per rank; 12 face cards J, Q, K) · numbers a to b: b − a + 1.

★ SPOT THE CLUE

Wording clues: "not" → rest of S · "at least one" → all − none · "exactly" → only that many · "or" → add, remove overlap · "neither … nor" → n(S) − n(A or B) · "not replaced" → update both counts · "between" → ends excluded.

✓ CHECK BEFORE YOU FINISH

Before finalising: (1) n(S) matches a multiplication or rule check. (2) Every favourable outcome passes the test, none is missed, none is counted twice. (3) n(E) on top, n(S) below. (4) 0 ≤ P ≤ 1. (5) Simplest form. (6) Complement check when easy.

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