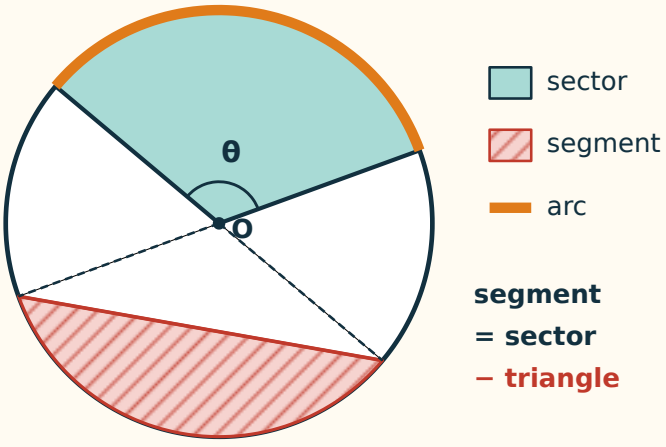
MATHS WITHOUT GUESSWORK

Areas Related to Circles

Understand the Diagram. Choose the Formula. Solve Independently.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27



Spot the region first. Then decide what to subtract or combine.

Core topics: area of a sector · length of an arc · area of a segment · shaded regions

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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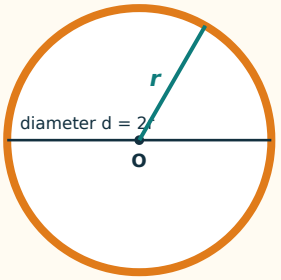
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Radius, diameter, circumference and area

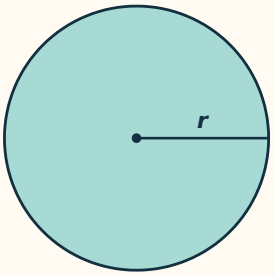
PREREQUISITE

Learning objective: Find the radius from any given measurement, and never confuse a boundary length with an area.



Circumference = 2πr (a length)

boundary vs inside



Area = πr² (a region)

◆ UNDERSTAND

The **radius** r joins the centre to the circle. The **diameter** d passes through the centre, so **d = 2r** and **r = d ÷ 2**. The **circumference** is the length of the boundary: C = 2πr = πd. The **area** is the amount of flat surface inside: A = πr².

➤ TWO DIFFERENT QUANTITIES

Small example (r = 7 cm, π = $\frac{22}{7}$):

C = 2 × $\frac{22}{7}$ × 7 = **44 cm**

A = $\frac{22}{7}$ × 7 × 7 = **154 cm²**

★ SPOT THE CLUE

Hidden radius. "Diameter 21 cm" → r = 10.5 cm. "Circumference 44 cm" → 2πr = 44 → r = 7 cm. "Largest circle in a square of side 12 cm" → the diameter equals the side, so r = 6 cm.

? WHY THIS STEP?

Why is the radius squared? Area counts unit squares, so it must be "length × length". Doubling the radius makes the circle twice as wide *and* twice as tall, so the area becomes 2 × 2 = 4 times as big. Circumference is a single length, so it only doubles.

⚠ COMMON TRAP

Using the diameter as r. With d = 14 cm, writing π × 14² gives 616 cm², four times too big. Always write "r = ..." as your first line.

If the question gives…	then r =	Example
the diameter d	d ÷ 2	d = 21 cm → r = 10.5 cm
the circumference C	C ÷ (2π)	C = 88 cm → r = 88 × 7 ÷ 44 = 14 cm
the area A	√(A ÷ π)	A = 154 cm² → r² = 49 → r = 7 cm
"largest circle in a square of side a"	a ÷ 2	a = 12 cm → r = 6 cm

✓ CHECK YOURSELF

C1. A circle has a diameter of 28 cm. Find (a) its circumference and (b) its area. Write the unit of each.

Method 2: area of a sector

CORE

Learning objective: Find a sector's area as a fraction of the whole circle, and explain why the formula works.

Hold this question: *If 360° gives the whole area πr², what does 1° give? What does θ give?*

◆ UNDERSTAND

Unitary method. The whole circle is a "sector" of angle 360°, with area πr². At 360° → πr². At 1° → $\frac{\pi r^2}{360}$. At θ → $\frac{\theta}{360} \times \pi r^2$.

➤ CHOOSE THE FORMULA

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

θ = central angle in degrees, r = radius. "The sector is $\frac{\theta}{360}$ of the whole circle."

SPOT THE REGION → SUBTRACT OR COMBINE

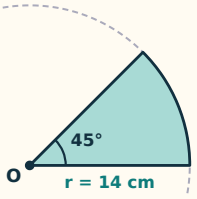
1 · SHADED REGION	A sector with central angle θ.
2 · FAMILIAR WHOLE	The full circle of the same radius.
3 · SUBTRACT / COMBINE	Take the fraction θ/360 of it (nothing removed yet).
REGION EQUATION	**sector = (θ/360) × circle**

WORKED EXAMPLE **r = 14 cm, θ = 45°**

Fraction = $\frac{45}{360} = \frac{1}{8}$

Area of circle = $\frac{22}{7} \times 14 \times 14 = 616$ cm²

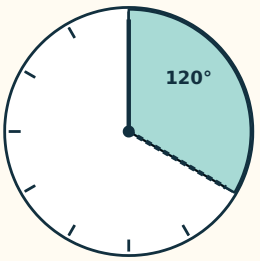
Area of sector = ⅛ × 616 = **77 cm²**



WORKED EXAMPLE **Minute hand 21 cm long, 20 minutes**

Angle turned = 20 × 6° = 120°, fraction = ⅓

Area swept = ⅓ × $\frac{22}{7}$ × 21 × 21 = ⅓ × 1386 = **462 cm²**



XII → IV: 20 minutes

? WHY THIS STEP?

Why find the whole circle first? It makes the fraction idea visible and keeps numbers simple: ⅛ of 616 is easier than multiplying four numbers together.

⚠ COMMON TRAP

Using the minutes as the angle. 20 minutes is not 20°. Convert first: 6° per minute.

✎ YOUR TURN

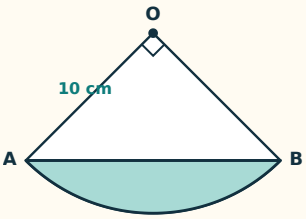
T2. The minute hand of a clock is 7 cm long. Find the area it sweeps in 15 minutes.

Method 4 (part 2): area of a segment, 90° and 60°

CORE

Learning objective: Find a segment as sector − triangle, writing the region equation first.

Hold this question: *The shaded region has a straight edge and a curved edge. What is the straight edge?*



SPOT THE REGION → SUBTRACT OR COMBINE

1 · SHADED REGION	Between chord AB and arc AB: a minor segment.
2 · FAMILIAR WHOLE	Sector OAB (two radii + arc).
3 · SUBTRACT / COMBINE	Subtract triangle OAB.
REGION EQUATION	**segment = sector OAB − ΔOAB**



WORKED EXAMPLE **r = 10 cm, ∠AOB = 90°, π = 3.14**

Sector = $\frac{90}{360} \times 3.14 \times 10 \times 10 = \frac{1}{4} \times 314 = 78.5$ cm²

ΔOAB = ½ × OA × OB = ½ × 10 × 10 = 50 cm² (OA ⊥ OB)

Segment = 78.5 − 50 = **28.5 cm²**

? WHY THIS STEP?

Why is ½ × OA × OB allowed? At 90°, OA is perpendicular to OB, so OB is the height when OA is the base. No other angle gives you this shortcut.

WORKED EXAMPLE **r = 21 cm, ∠AOB = 60°, √3 = 1.73**

Sector = $\frac{60}{360} \times \frac{22}{7} \times 21 \times 21 = 231$ cm²

ΔOAB is equilateral (side 21): $\frac{\sqrt{3}}{4} \times 21^2 = \frac{441\sqrt{3}}{4} = \frac{441 \times 1.73}{4} = 190.7325$ cm²

Segment = 231 − 190.7325 = 40.2675 ≈ **40.27 cm²**

(Exact form, if asked: $(231 - \frac{441\sqrt{3}}{4})$ cm².)

✔ CHECK YOURSELF

Is it sensible? A 60° segment is a thin sliver: 40.27 is about 17% of the sector (231). A segment can never be larger than its sector.

⚠ COMMON TRAP

Stopping at the sector. If the diagram shows a chord, the triangle must be removed. "Sector area" is not the answer to a segment question.

✎ YOUR TURN

T6. A chord of a circle of radius 14 cm subtends a right angle at the centre. Find the area of the minor segment.

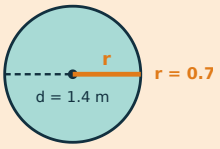
Doubts 1 and 2: radius and "which quantity?"

DOUBT 1 *"The formula says πr², but the question gave me the diameter. Why can't I just use that number?"*

✗ WHY IT HAPPENS
The formula uses whatever letter is in it, and students substitute whatever length the question offers.

◆ THE CONCEPT
r is the distance from the **centre** to the circle. πr² counts about 3.14 squares of side r. Squares of side d = 2r are four times larger, so using d makes the answer four times too big.

★ WORKED EXAMPLE
A circular table top has diameter 1.4 m. r = 1.4 ÷ 2 = 0.7 m. Area = $\frac{22}{7}$ × 0.7 × 0.7 = **1.54 m²**.



? WHY THIS STEP?
Halving first is forced by the definition of r. Writing "r = 0.7 m" as a separate line shows the examiner you know it.

⚠ WRONG APPROACH
$\frac{22}{7}$ × 1.4 × 1.4 = 6.16 m², exactly 4 times too much.

✎ YOUR TURN DB1
A circle has diameter 42 cm. Find its area.

✓ REMEMBER
Write **"r = ..."** before every formula.

DOUBT 2 *"Some questions say "how much wire" and some say "how much cloth". How do I know whether to find area or circumference?"*

✗ WHY IT HAPPENS
Both formulas contain π and r, so students react to the *circle* instead of to the *question*.

◆ THE CONCEPT
Ask: could I lay the answer along a line (a **length**: wire, fence, rim, border) or spread it over a surface (an **area**: cloth, paint, grass)? Lengths use 2πr; areas use πr².

★ WORKED EXAMPLE
Circular park, r = 21 m. Fencing = 2 × $\frac{22}{7}$ × 21 = **132 m**. Grass to cover it = $\frac{22}{7}$ × 21 × 21 = **1386 m²**.



? WHY THIS STEP?
The fence goes round the boundary only; the grass covers every part of the surface inside.

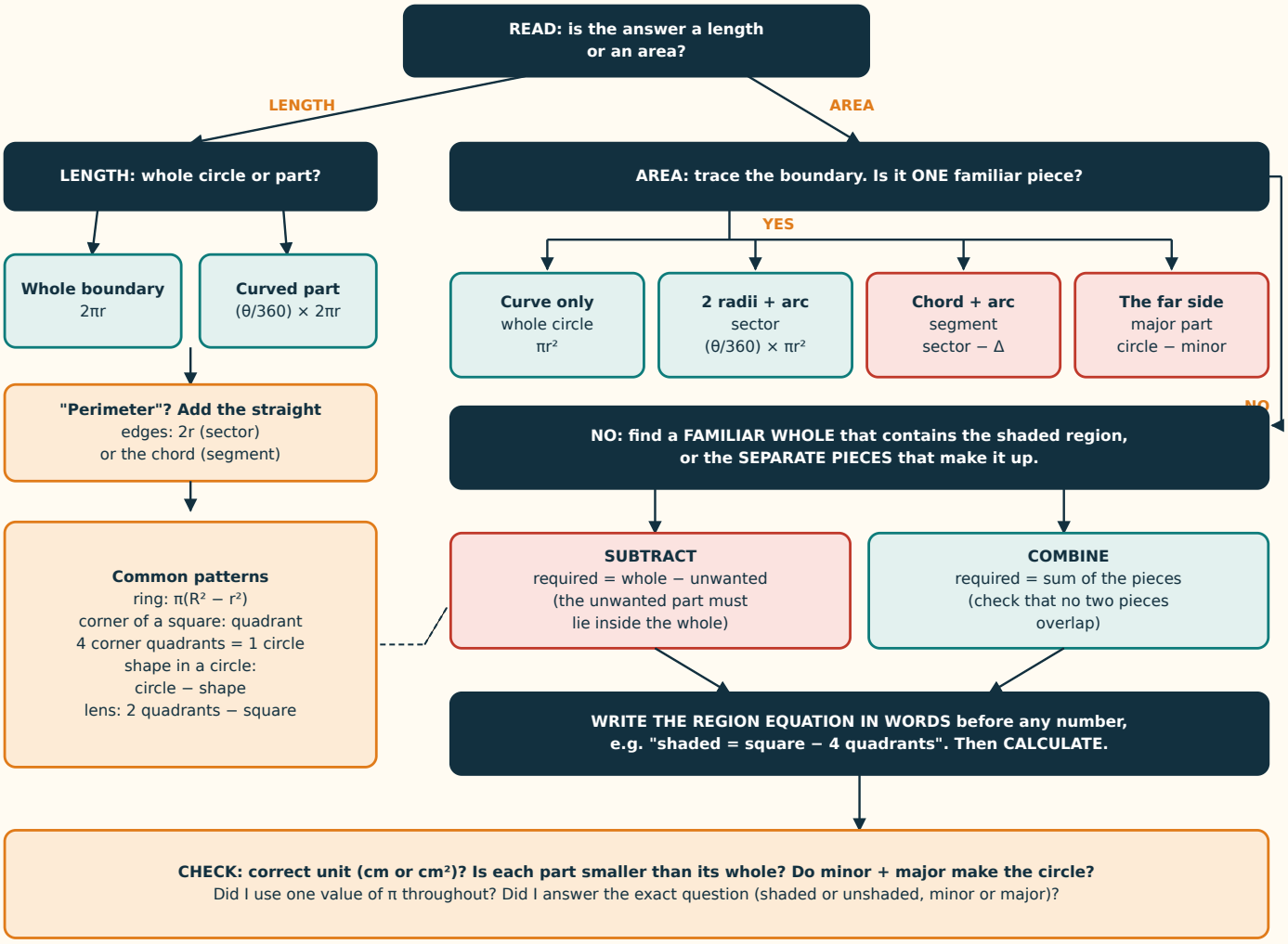
⚠ WRONG APPROACH
Answering "1386 m of fencing". The unit would be m², which cannot measure a fence.

✎ YOUR TURN DB2
A circular park has radius 35 m. Find (a) the length of fencing around it, (b) the area to be planted with grass.

✓ REMEMBER
Length → cm, m. Area → cm², m². The unit tells you which formula.

Decision tree: what should I calculate first?

Learning objective: Follow the branches from the question to your first calculation, and know what to check at the end.



Navy = a question to ask · teal = a single formula · coral = subtract · orange = reminders and checks

★ SPOT THE CLUE

Run the tree once. "A chord of a circle of radius 14 cm subtends 90° at the centre. Find the area of the minor segment." → AREA → one familiar piece? Yes: chord + arc → **segment = sector − ΔOAB** → write it in words → 154 − 98 = 56 cm² → CHECK: cm² ✓, 56 < 154 ✓.

? WHY THIS STEP?

Why does the tree start with "length or area"? It instantly halves the choices and fixes the unit of your answer. Students who skip it are the ones who give a fence in m².

⚠ COMMON TRAP

A tree is a guide, not a machine. Some questions need both branches: "find the area of the segment and the length of its boundary". Run the tree once for each part.

Mistake clinic (1 of 3): reading the circle

MISTAKE 1 · Diameter used as the radius

✗ STUDENT'S WORK	*Area of a circle of diameter 14 cm.* "$\frac{22}{7} \times 14 \times 14 = 616$ cm²"
⚠ WHERE IT FAILS	14 cm is the diameter. The formula needs r = 7 cm, so the answer is 4 times too big.
✓ CORRECTED	r = 14 ÷ 2 = 7 cm. Area = $\frac{22}{7} \times 7 \times 7 = 154$ cm².
➤ PREVENTION RULE	Write "r = …" as your first line.
✎ FIX IT E1	Find the area of a circle of diameter 35 cm.

MISTAKE 2 · Circumference used instead of area

✗ STUDENT'S WORK	*Area of a circle of radius 7 cm.* "$2 \times \frac{22}{7} \times 7 = 44$ cm²"
⚠ WHERE IT FAILS	2πr is the boundary length (in cm), not the surface inside. A length cannot be measured in cm².
✓ CORRECTED	Area = πr² = $\frac{22}{7} \times 7 \times 7 = 154$ cm².
➤ PREVENTION RULE	An area formula always contains r² and gives square units.
✎ FIX IT E2	Find the area of a circle of radius 21 cm.

MISTAKE 3 · Radius not squared

✗ STUDENT'S WORK	*Sector of radius 14 cm, angle 90°.* "¼ × $\frac{22}{7} \times 14 = 11$ cm²"
⚠ WHERE IT FAILS	The radius appears only once, so this multiplies a length by a number: it cannot be an area.
✓ CORRECTED	¼ × $\frac{22}{7} \times 14 \times 14 = ¼ \times 616 = 154$ cm².
➤ PREVENTION RULE	Write r × r in full rather than r².
✎ FIX IT E3	Find the area of a sector of radius 7 cm and angle 60°.

MISTAKE 4 · Wrong central angle

✗ STUDENT'S WORK	*Area swept by a 21 cm minute hand in 10 minutes.* "(10/360) × 1386 = 38.5 cm²"
⚠ WHERE IT FAILS	10 minutes is not 10°. The minute hand turns 6° every minute.
✓ CORRECTED	Angle = 10 × 6° = 60°. Area = ⅙ × 1386 = 231 cm².
➤ PREVENTION RULE	Convert time to degrees first: 6° per minute.
✎ FIX IT E4	Find the area swept by a minute hand 14 cm long in 35 minutes.

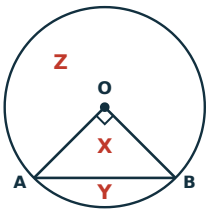
Practice 1: recognise

Learning objective: Name regions, choose the quantity, and find the radius and angle before calculating anything.

1 Recognise | 2 Apply | 3 Connect | 4 Board-style

No calculation needed for most. Answers: Section H.

P1 In the figure, O is the centre and ∠AOB = 90°. X is ΔOAB, Y is the region between chord AB and the minor arc, and Z is the rest of the circle. Name: (a) X, (b) Y, (c) X and Y together, (d) Z, (e) X and Z together.



Purpose: Name regions by their boundaries. **easy ★☆☆**

Working space (use separate paper if needed)

P2 Length or area? Say which, and which formula: (a) wire for a circular photo frame; (b) cloth for a round tablecloth; (c) the distance moved by the tip of a minute hand in 15 minutes; (d) the region cleaned by a wiper; (e) fencing for a semicircular garden.

Purpose: Separate boundary questions from region questions. **easy ★☆☆**

Working space (use separate paper if needed)

P3 Find the radius: (a) diameter 18 cm; (b) circumference 66 cm; (c) the largest circle inside a square of side 12 cm; (d) the circle passing through all four corners of a square of side 10 cm.

Purpose: Find hidden radii. **moderate ★★☆**

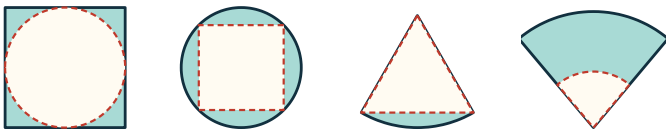
Working space (use separate paper if needed)

P4 Find the angle at the centre: (a) a minute hand in 25 minutes; (b) an hour hand in 2 hours; (c) each of 10 equal sectors; (d) the major sector when the minor sector is 75°.

Purpose: Turn situations into central angles. **easy ★☆☆**

Working space (use separate paper if needed)

P5 Write the region equation in words for each shaded (teal) region. No numbers.



(a) (b) (c) (d)

Purpose: Spot the region, then subtract. **moderate ★★☆**

Working space (use separate paper if needed)

P6 True or false? Give a reason. (a) The area of a semicircle is ½πr². (b) The perimeter of a semicircle is πr. (c) Segment = sector + triangle. (d) Doubling the radius doubles the area.

Purpose: Catch common misconceptions. **moderate ★★☆**

Working space (use separate paper if needed)

Revision sheet

Learning objective: Recall every formula, triangle and region pattern from one page.

Quantity	Formula	Remember
Circle	$A = \pi r^2$, $C = 2\pi r = \pi d$	Find r first: $r = d \div 2$.
Sector (angle θ)	$(\theta/360) \times \pi r^2$	Two radii + arc. Semicircle $\frac{1}{2}\pi r^2$, quadrant $\frac{1}{4}\pi r^2$.
Arc	$(\theta/360) \times 2\pi r$	A length, in cm.
Perimeter	sector: arc + 2r; segment: arc + chord	Include every straight edge.
Segment (minor)	sector − ΔOAB	Syllabus angles only: 60°, 90°, 120°.
Major sector / segment	πr^2 − minor part	Or $(360° - \theta)/360 \times \pi r^2$ for the major sector.

➤ THE THREE TRIANGLES

∠AOB	chord AB	area ΔOAB
60°	r	$(\sqrt{3}/4)\ r^2$
90°	$r\sqrt{2}$	$\frac{1}{2}\ r^2$
120°	$r\sqrt{3}$	$(\sqrt{3}/4)\ r^2$

◆ VALUES AND UNITS

π: $\frac{22}{7}$ unless told; 3.14 if told; keep π if "in terms of π".

Clocks: minute hand 6°/min; hour hand 30°/hour. **Units:** lengths in cm, areas in cm^2; 1 m^2 = 10 000 cm^2. **Rounding:** only at the end.

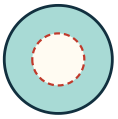
THE EIGHT REGION PATTERNS



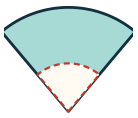
Segment
sector − triangle



Major sector
circle − minor sector



Ring
$\pi(R^2 - r^2)$



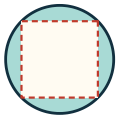
Part of a ring
big sector − small sector



Corners
square − circle



Corner quadrants
square − 4 quadrants



Around a square
circle − square



Lens
2 quadrants − square

In every case: ① spot the shaded region ② name the familiar whole (or pieces) ③ subtract the unwanted part, or add the pieces without overlap ④ write the region equation in words, **then** calculate.

1 READ | 2 IDENTIFY | 3 VISUALISE | 4 CHOOSE | 5 CALCULATE | 6 SUBTRACT / COMBINE | 7 CHECK

⚠ COMMON TRAP

Final error checklist. ☐ d used as r (E1) ☐ 2πr for area (E2) ☐ r not squared (E3) ☐ minutes as degrees (E4) ☐ sector given as segment (E5) ☐ added a hole (E6) ☐ wrong whole (E7) ☐ mixed units (E8) ☐ missing ² (E9) ☐ early rounding (E10) ☐ wrong π (E11) ☐ skipped steps (E12)

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- **Core methods.** Worked examples with a reason for every step, and quick checks to try.
- **Doubt Bank.** 16 common doubts answered clearly, each followed by a question to try.
- **Method selection.** Clue tables, decision charts and look-alike questions that train you to choose the method.
- **Mistake Clinic.** 12 common mistakes: the wrong work, why it is tempting, and the fix.

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