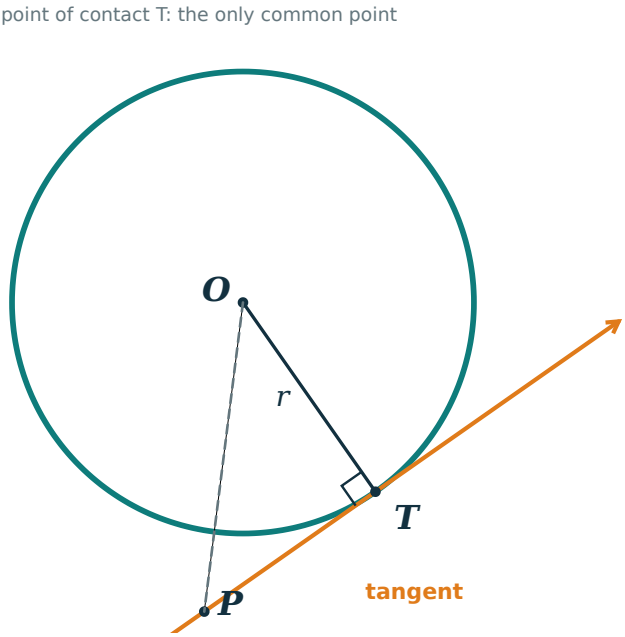
MATHS WITHOUT GUESSWORK

Circles

Understand the Diagram.
Recognise the Theorem.
Solve Independently.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27



OT ⊥ PT: the radius meets the tangent at 90°.

INSIDE THIS BOOK

Read a circle diagram
Spot the clue
Check a theorem's conditions
Write a proof with reasons
Solve new questions alone

Signature feature:
Why does this theorem apply here?

Core topics: tangent to a circle at the point of contact · tangent ⊥ radius · equal tangents from an external point

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

Liked this? The full Circles workbook is ₹99 →

Contents

How to use: Tap any line to jump to that page. Page numbers match the numbers printed on each page.

A Orientation 2–5

Welcome 2
How to use this workbook 3
The routine and the signature feature 4
Diagnostic challenge 5

B Foundation 6–10

Circle words that matter here 6
A line and a circle: three possibilities 7
Your geometry toolkit 8
Can you read the diagram? 9
The chapter in one picture 10

C Core Theorems 11–21

Concept 1 · Understanding a tangent 11
Theorem 1 · Tangent ⊥ radius at the point of contact 12
Using Theorem 1 to find lengths 13
Theorem 1 in angles and proofs 14
Concept 3 · How many tangents from a point? 15
Theorem 2 · Tangents from an external point are equal 16
What else Theorem 2 tells you 17
Equal tangents in polygons around a circle 18
Concept 5 · Connecting the theorems (1) 19
Concept 5 · Connecting the theorems (2) 20
Same diagram, new questions; new diagram, same theorem 21

D Doubt Bank 22–28

Doubts 1–2: tangent or secant? 22
Doubts 3–4: why 90°, which radius? 23
Doubts 5–6: radii and counting 24
Doubts 7–8: which segments, which theorem? 25
Doubts 9–10: starting a proof 26
Doubts 11–12: RHS, and the figure 27
Doubts 13–14: new figures, exam reasons 28

E Theorem Selection 29–32

Which theorem? Check the conditions 29
Theorem-selection flowchart 30
Tempting, but not allowed 31
Choose the theorem first 32

F Mistakes & Proof Writing 33–37

Mistake clinic (1 of 3) 33
Mistake clinic (2 of 3) 34
Mistake clinic (3 of 3) 35
Writing a board-exam proof 36
Finish the proof; spot the wrong step 37

G Practice 38–43

Practice 1 · Recognise 38
Practice 2 · Apply (1) 39
Practice 3 · Apply (2) 40
Practice 4 · Connect 41
Practice 5 · Board-style challenge 42
Self-test · timed 43

H Answer Key 44–47

Answer key and full explanations 44–47

I Revision 48–49

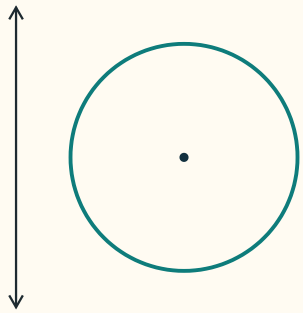
One-page revision sheet 48
My progress tracker 49

A line and a circle: three possibilities

CORE

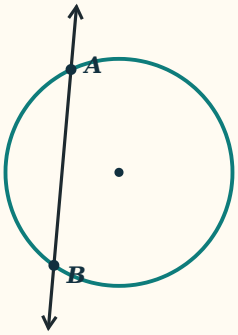
Learning objective: Classify a line as non-intersecting, secant or tangent, and define the point of contact.

Hold this question: *How many points can a straight line and a circle share, and why does "exactly one" matter so much?*



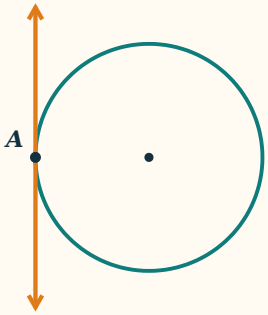
Non-intersecting line

(i) no common point



Secant

(ii) two common points

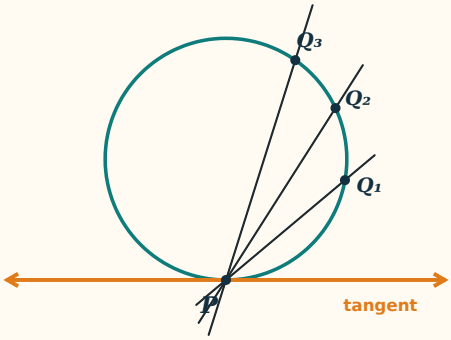


Tangent

(iii) exactly one common point

A line and a circle in the same plane share **0, 1 or 2 points**, never 3: a straight line can enter a circle once and leave once. The three cases have names:

- **Non-intersecting line**: no common point.
- **Secant**: exactly two common points (it cuts the circle; the part inside is a chord).
- **Tangent**: exactly one common point. That point is the **point of contact**, and the tangent is said to **touch** the circle there.



Turn a secant about P: Q slides towards P.

◆ UNDERSTAND

A tangent is the last position of a secant. Fix P on the circle and turn the line PQ about P. The second point Q slides along the circle towards P. When Q reaches P, the two common points have become one: the line is now the tangent at P. Turning a little further gives a secant again, so **at each point of a circle there is exactly one tangent**.

★ SPOT THE CLUE

The words **"touches"**, **"tangent at T"**, **"point of contact"**, and **"circle inscribed in / polygon circumscribing a circle"** (every side touches the circle) all tell you a line is a tangent. "Cuts", "meets at A and B", or "intersects" with two points named tell you it is a secant.

⚠ COMMON TRAP

Deciding from the picture. A secant that cuts very close to the edge can *look* like a tangent. Use only what the question states (or what you prove). Diagrams in exams are not drawn to scale.

✓ CHECK YOURSELF

C2. Classify each line and give a reason: (a) line *a* meets the circle at X and Y; (b) line *b* touches the circle at Z; (c) line *c* passes through the centre O. Then say why no line can meet a circle in three points.

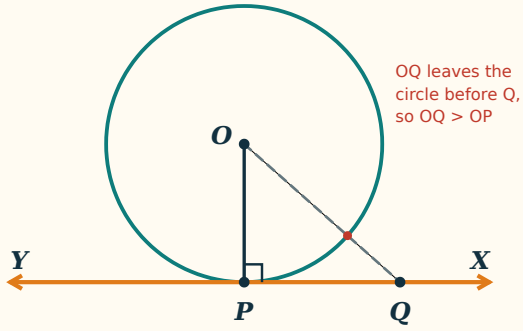
Theorem 1 · Tangent ⊥ radius at the point of contact

CORE

Learning objective: Explain why Theorem 1 is true, prove it, and state exactly when it applies.

The situation. On a rolling bicycle wheel, the spoke that points to the ground always stands straight up from the road. The spoke is a radius; the road is a tangent; they meet at 90°.

Why it makes sense. Stand at the centre O and look at the tangent line. Every point of the tangent except P is outside the circle, so it is *farther* from O than r. P is the only point at distance exactly r. So P is the **nearest** point of the line to O, and the shortest path from a point to a line is the perpendicular one.



XY touches the circle at P; Q is any other point of XY.

> **Theorem 1.** The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given: a circle with centre O and a tangent XY touching it at P. **To prove:** OP ⊥ XY.

#	Statement	Reason
1	Take any point Q on XY other than P, and join OQ.	Construction, to compare OQ with OP.
2	Q lies outside the circle.	If Q were inside or on the circle, XY would meet the circle at a second point and be a secant, not a tangent.
3	So OQ > r, that is, OQ > OP.	A point outside is farther from O than the radius; OP = r because P is on the circle.
4	This is true for **every** point Q of XY except P.	Q was any point other than P.
5	So OP is the shortest distance from O to the line XY.	From rows 3 and 4.
6	Hence OP ⊥ XY. ∎	Of all segments from a point to a line, the perpendicular is the shortest.

★ **WHY DOES THIS THEOREM APPLY HERE?** Theorem 1

Condition the theorem needs	Here?	How I know (evidence)
The line is a **tangent** (exactly one common point)	✓ **yes**	stated in the question, or proved
The radius goes to the **point of contact** of that tangent	✓ **yes**	the radius must end where the tangent touches

Then and only then: the radius and the tangent meet at 90°. A radius to any other point of the circle gets **no** right angle from this theorem.

◆ **UNDERSTAND**

In practice the theorem hands you a right angle at every point of contact. A right angle means a right triangle, and a right triangle means Pythagoras, RHS congruence and angle facts become available. Remark: since the perpendicular to OP at P is unique, there is exactly one tangent at each point of a circle.

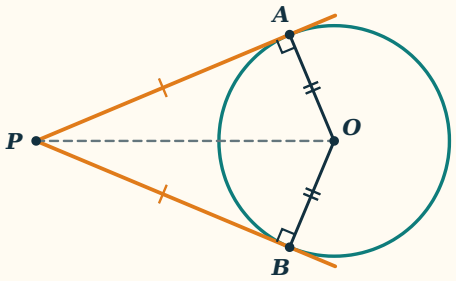
Theorem 2 · Tangents from an external point are equal

CORE

Learning objective: Prove that PA = PB, naming the correspondence and the congruence rule.

The situation. From a point P outside a circle, draw both tangents. Measure PA and PB: they always match. Why? The whole figure is **symmetrical about the line OP**. Fold the page along OP and the circle lands on itself, so the tangent PA lands exactly on PB.

A fold is a good idea, not a proof. The proof turns the symmetry into two congruent right triangles.



PA, PB touch the circle at A, B. OA, OB, OP joined.

Theorem 2. The lengths of tangents drawn from an external point to a circle are equal.

Given: a circle with centre O; P is outside; PA and PB are tangents touching the circle at A and B. **To prove:** PA = PB. **Construction:** join OA, OB and OP.

#	Statement	Reason
1	∠OAP = 90°	OA is the radius through the point of contact of tangent PA (Theorem 1)
2	∠OBP = 90°	OB is the radius through the point of contact of tangent PB (Theorem 1)
3	In right triangles OAP and OBP: OA = OB	radii of the same circle
4	OP = OP, and OP is the hypotenuse of both	common side; it is opposite the right angle in each
5	ΔOAP ≅ ΔOBP, with A ↔ B, O ↔ O, P ↔ P	RHS: right angle, hypotenuse, one side (rows 1–4)
6	PA = PB ∎	CPCT: PA and PB are corresponding sides

★ **WHY DOES THIS THEOREM APPLY HERE?** RHS congruence inside Theorem 2

Condition the theorem needs	Here?	How I know (evidence)
Both triangles are right-angled	✓ **yes**	∠OAP = ∠OBP = 90°, rows 1–2, from Theorem 1
Their hypotenuses are equal	✓ **yes**	both hypotenuses are OP (common)
One other pair of sides is equal	✓ **yes**	OA = OB, radii of the same circle

All three RHS conditions hold, so the triangles are congruent. **Why not SAS?** The known angle (90° at A) lies between OA and AP, and AP is the unknown. SAS needs the angle *between* the two known sides, so it does not apply.

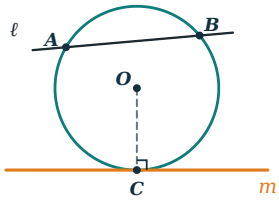
◇ **FIND THE MISSING LINK**

Rows 1 and 2 are the bridge from Theorem 1 to Theorem 2. Remove them and RHS has no right angle to use. That is why Theorem 1 must be proved first.

Doubts 1–2: tangent or secant?

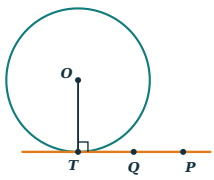
DOUBT 1 *"Two lines in the figure both seem to touch the circle. How do I know which one is the tangent?"*

✗ WHY THIS CONFUSION HAPPENS	A drawing is never exact. A secant that cuts near the edge can look as if it only touches.
◆ UNDERSTAND	Count the common points: a **tangent** has exactly one, a **secant** has exactly two. Decide from the wording (or a proof), never from the look.
★ SPOT THE CLUE	"touches at C", "tangent at C", "point of contact" → tangent. "meets the circle at A and B", "cuts", two named points → secant.
➤ CHOOSE THE THEOREM	No theorem yet: use the **definition**. Only after a line is known to be a tangent may Theorems 1 and 2 be used on it.
✦ SOLVED EXAMPLE	Line ℓ meets the circle at A and B; line m touches it at C. ℓ has two common points, so it is a **secant**. m has one, so it is a **tangent**, and OC $\perp$ m (Theorem 1). Nothing makes OA $\perp$ ℓ.
? WHY THIS STEP?	Classifying first protects every later step: a right angle claimed on a secant is simply false.
⚠ COMMON TRAP	"ℓ looks as if it just grazes the circle, so OA $\perp$ ℓ." Wrong: ℓ meets the circle twice.
✎ YOUR TURN DB1	Line a meets a circle at exactly one point P. Line b passes through P and the centre O. Classify both lines, with reasons.
✓ CHECK YOURSELF	A line through the centre always contains a diameter, so it meets the circle twice.



DOUBT 2 *"What exactly is the point of contact? Is it any point on the tangent?"*

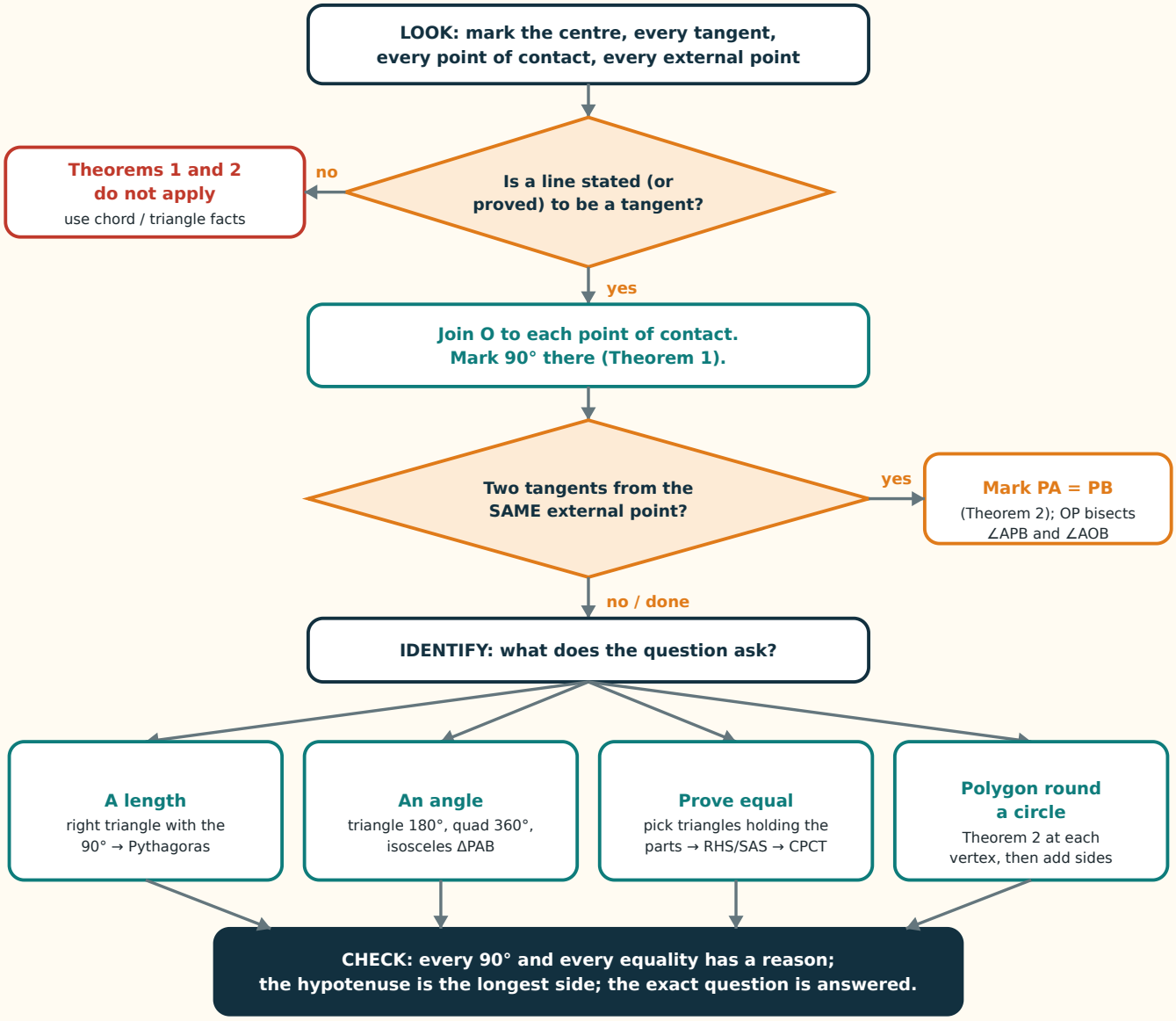
✗ WHY THIS CONFUSION HAPPENS	A tangent line has endless points. It is easy to put the right angle in the wrong place.
◆ UNDERSTAND	The point of contact is the **one** point the tangent shares with the circle. Every other point of the tangent is outside the circle, at distance more than r from O.
★ SPOT THE CLUE	"touches the circle at T": T is the point of contact. In a figure it is where the tangent meets the curve.
➤ CHOOSE THE THEOREM	Theorem 1 puts its 90° **at the point of contact**, between the tangent and the radius to that point.
✦ SOLVED EXAMPLE	PT touches at T and Q is another point of line PT. The right angle is ∠OTP (at T). ∠OQP is **not** 90°: OQ is not a radius (Q is outside the circle).
? WHY THIS STEP?	Finding T first tells you where the right angle and the right triangle are.
⚠ COMMON TRAP	Putting the square at P, the external point, or at Q. The tangent is perpendicular to OT only.
✎ YOUR TURN DB2	XY touches a circle with centre O at A. B is another point on XY. Is OB greater than, equal to, or less than OA? Why?
✓ CHECK YOURSELF	Only one point of the tangent is at distance exactly r from O.



Theorem-selection flowchart

Learning objective: Follow a decision path from the figure to the first correct step.

Follow the arrows from the top. Every branch ends in an action you already know. The skill is choosing the branch, and each diamond is a condition check.



⚠ COMMON TRAP

A flowchart is a guide, not a machine. Many questions go down **more than one** branch: the chord-and-two-tangents problem on page 20 needs a length, an angle bisector and a right triangle. Use the chart to find the first step, then look again.

⚠ COMMON TRAP

Choosing a theorem because the figure “looks like” the textbook figure. Two tangents drawn from *different* points can look exactly like the Theorem 2 picture, and Theorem 2 does not apply to them.

✓ CHECK YOURSELF

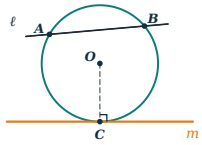
Try the chart now. “PA and PB are tangents from P; ∠AOB = 116°; find ∠APB.” Which boxes do you pass through? (Answer: tangent → join radii, 90° → same external point → an angle → quadrilateral 360°, giving 64°.)

Mistake clinic (1 of 3)

MISTAKE 1 · Treating a secant as a tangent

✗ STUDENT'S WORK

"Line l meets the circle at A and B. OA is a radius, so OA ⊥ l (radius ⊥ tangent)."

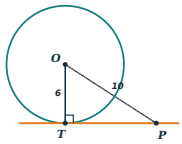


⚠ WHERE IT FAILS	*l* has two common points, so it is a secant. Theorem 1 is about tangents only.
? WHY TEMPTING	The words "radius" and "line" are both there, and the theorem is fresh in memory.
✓ CORRECTED	*l* is a secant; no right angle at A is known.
➤ PREVENTION RULE	**Before using Theorem 1, count the common points: exactly one.**
✎ FIX IT E1	Line *m* passes through P on a circle and through Q inside it. A student writes OP ⊥ *m*. Explain the error.

MISTAKE 2 · Right angle at the wrong vertex, wrong hypotenuse

✗ STUDENT'S WORK

"PT touches at T; OT = 6, OP = 10. ∠OPT = 90°, so $OT^2 = OP^2 + PT^2$, $PT^2 = 36 - 100$."

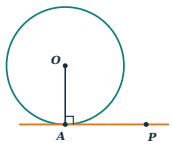


⚠ WHERE IT FAILS	The right angle is at T, the point of contact, so the hypotenuse is OP, not OT.
? WHY TEMPTING	The 90° sign is placed at the letter P that appears in the question.
✓ CORRECTED	∠OTP = 90° (Theorem 1). $PT^2 = OP^2 - OT^2 = 100 - 36 = 64$, PT = 8.
➤ PREVENTION RULE	**Right angle at the point of contact; hypotenuse runs from O to the external point.**
✎ FIX IT E2	PA touches a circle at A. OA = 10 cm, OP = 26 cm. A student writes $PA^2 = OA^2 + OP^2$. Correct it and find PA.

MISTAKE 3 · Using an arbitrary radius

✗ STUDENT'S WORK

"The tangent at A … OB is a radius, so OB ⊥ the tangent."



⚠ WHERE IT FAILS	B is not the point of contact. Only OA is perpendicular to the tangent at A.
? WHY TEMPTING	"All radii are equal" is stretched into "all radii behave the same".
✓ CORRECTED	OA ⊥ tangent at A. For angles at B, use ΔOAB with OA = OB.
➤ PREVENTION RULE	**Follow the radius to its end point: is that the point of contact?**
✎ FIX IT E3	The tangent at C meets a circle with centre O; D is another point of the circle. Which of OC, OD is ⊥ to the tangent? Why?

Practice 1 · Recognise

Learning objective: Identify tangents, secants, points of contact and the clues that point to each theorem.

1 Recognise | 2 Apply | 3 Connect | 4 Board-style

Answers: Section H

P1 A line meets a circle at exactly two points. The line is called
(A) a tangent (B) a secant (C) a radius (D) a non-intersecting line

Purpose: Recognise a secant. **easy ★☆☆**

Working space (use separate paper if needed)

P2 How many tangents can be drawn to a circle through a point that lies on the circle?
(A) 0 (B) 1 (C) 2 (D) infinitely many

Purpose: Count tangents from a position. **easy ★☆☆**

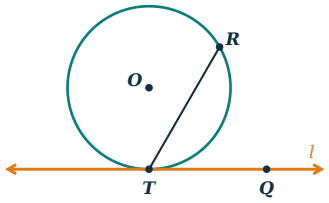
Working space (use separate paper if needed)

P3 Fill in the blanks. (a) A tangent and its circle have ____ common point(s). (b) That common point is called the ____. (c) Tangents at the two ends of a diameter are ____. (d) At most ____ tangents of a circle can be parallel to a given line.

Purpose: Use the vocabulary precisely. **easy ★☆☆**

Working space (use separate paper if needed)

P4 Line *l* touches the circle (centre O) at T; Q is on *l*; R is on the circle. (a) Name the point of contact. (b) Which radius is perpendicular to *l*? (c) Is ∠OQT = 90°? (d) Is TR a tangent? Give a reason for each.



Purpose: Read a diagram with reasons. **easy ★☆☆**

Working space (use separate paper if needed)

P5 A circle has radius 7 cm. How many tangents to it pass through a point at distance (a) 5 cm, (b) 7 cm, (c) 9.5 cm from the centre?

Purpose: Apply the position test. **easy ★☆☆**

Working space (use separate paper if needed)

P6 True or false? Give a reason. (a) Every line through a point outside a circle is a tangent. (b) The radius to any point of a circle is perpendicular to every tangent. (c) The two tangents from an external point are equal in length. (d) A tangent can pass through the centre.

Purpose: Spot over-generalisations. **moderate ★★☆**

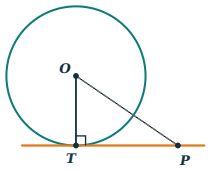
Working space (use separate paper if needed)

One-page revision sheet

Learning objective: Recall every definition, both theorems with their conditions, and the clues that point to them.

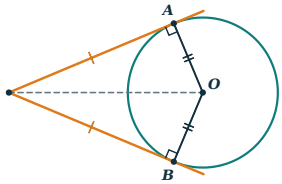
◆ UNDERSTAND

Tangent: a line meeting the circle at exactly one point, the **point of contact**. **Secant:** a line meeting it at two points. **Length of a tangent** from an external point P: PT, from P to the point of contact. **Tangents through P:** inside 0 · on the circle 1 · outside 2 (test: compare OP with r).



Theorem 1 CORE · PROVE

The tangent at any point of a circle is ⊥ to the radius through the point of contact.
Needs: a tangent; the radius to *its* contact point.
Proof idea: every other point of the tangent is outside, so OP is the shortest distance, hence ⊥.



Theorem 2 CORE · PROVE

Tangents from an external point are equal: PA = PB.
Needs: both tangents from the same point.
Proof: 90° at A, B (Thm 1); OA = OB; OP common; RHS; CPCT.

Diagram-reading clues

If you see…	…do this
“touches”, “tangent at T”, “point of contact”	join O to T; mark 90° (Theorem 1)
tangent + centre + a length	right triangle; hypotenuse = O to external point
two tangents from one point P	PA = PB; OP bisects ∠APB and ∠AOB
angle at P or at O between two tangents	∠APB + ∠AOB = 180°
polygon around a circle	two equal tangents at every vertex
chord of a big circle touching a smaller concentric one	point of contact is the chord’s midpoint

★ THE FOUR-QUESTION CHECK — before you write any step

Condition the theorem needs	Here?	How I know (evidence)
Is the line really a tangent?	☐	stated or proved, never “it looks like it”
Does the radius end at its point of contact?	☐	otherwise there is no right angle
Do both tangent segments start at one point?	☐	otherwise there is no equality
Is each 90° and each equality backed by a reason?	☐	given, a theorem, or proved

Four ticks → use the theorem. One “no” → choose a different fact.

⚠ COMMON TRAP

Final error checklist. ☐ secant treated as tangent (E1) ☐ right angle at the wrong vertex (E2) ☐ arbitrary radius (E3) ☐ 90° assumed (E4) ☐ tangents from different points (E5) ☐ SAS instead of RHS (E6) ☐ missing reasons (E7) ☐ right number, missing reasons (E8) ☐ measuring the figure (E9)

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