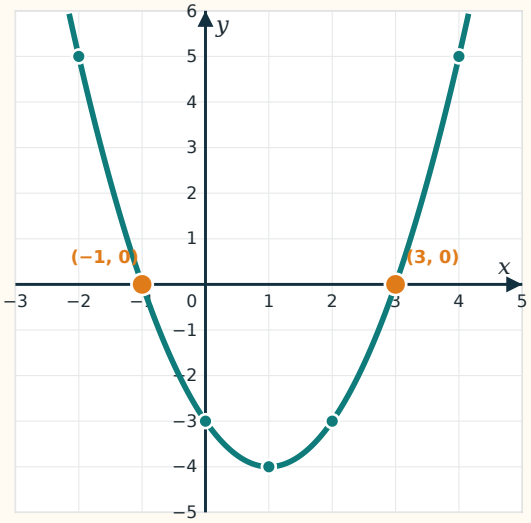
MATHS WITHOUT GUESSWORK

Polynomials

Understand the Graph.
Recognise the Relationship.
Solve Independently.

A step-by-step learning and practice workbook for Class 10 Mathematics

CBSE Class 10 Mathematics · Syllabus session 2026–27



WHY IT WORKS

a = 1, b = −2

c = −3

−b/a = 2

c/a = −3

Section C explains why.

$p(x) = x^2 - 2x - 3$

zeroes −1 and 3 · sum 2 · product −3

Core topics: zeroes of a polynomial · zeroes and graphs · zeroes and coefficients of a quadratic

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

Liked this? The full Polynomials workbook is ₹99 →

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PREREQUISITE

Evaluating a polynomial: the value p(k)

Learning objective: Substitute a number into a polynomial correctly, keeping every sign.

◆ UNDERSTAND

p(x) is a rule: put in a number, get out a number. **p(2)** means “replace every x by 2 and work out the result”. The answer is a **value**: a single number.

➤ CHOOSE THE METHOD

To find p(k): (1) write p(x); (2) replace each x by k **inside brackets**; (3) work out powers first; (4) then multiplications; (5) then add and subtract from left to right.

Example. $p(x) = x^2 - 5x + 6$, find $p(-2)$.
$p(-2) = (-2)^2 - 5(-2) + 6 = 4 + 10 + 6 = 20$. **$p(-2) = 20$.**

More values of $p(x) = 2x^2 - 3x + 1$:

x	Substitute with brackets	Value p(x)
2	$2(2)^2 - 3(2) + 1 = 8 - 6 + 1$	**3**
−1	$2(-1)^2 - 3(-1) + 1 = 2 + 3 + 1$	**6**
0	$2(0)^2 - 3(0) + 1 = 0 - 0 + 1$	**1**
½	$2(½)^2 - 3(½) + 1 = ½ - 3/2 + 1$	**0**

Look at the last row: the value is 0. That is special. We meet it on the next page.

? WHY THIS STEP?

Why brackets? $(-2)^2$ means $(-2) \times (-2) = 4$. Without brackets, -2^2 means $-(2 \times 2) = -4$. Brackets keep the sign *inside* the power. They also stop you losing the sign in $5(-2) = -10$, so $-5(-2) = +10$.

⚠ COMMON TRAP

Sign check. A student writes $p(-2) = -2^2 - 5(-2) + 6 = -4 + 10 + 6 = 12$. The first term is wrong: $(-2)^2 = 4$, not -4. The correct value is 20 (see above). Always bracket a negative input.

✓ CHECK YOURSELF

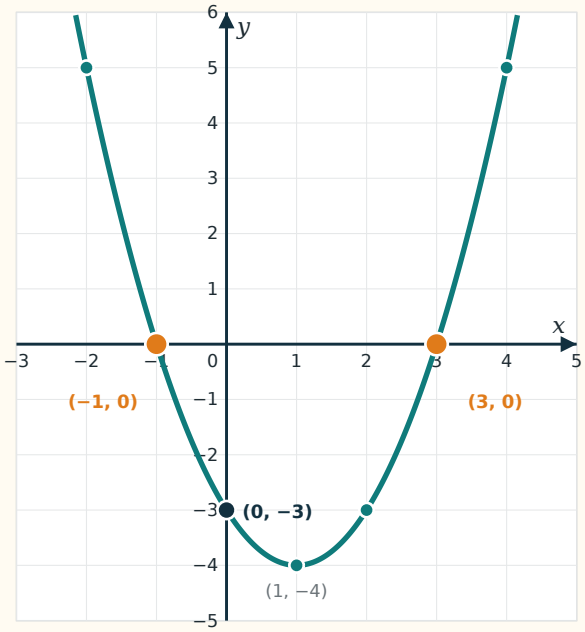
C2. (a) For $p(x) = x^2 + 4x - 2$, find $p(-3)$. (b) For $q(x) = 3x^2 - 2x$, find $q(0)$ and $q(⅓)$.

CORE

The graph connection

Learning objective: Explain why the zeroes of p(x) are the x-coordinates where the graph of y = p(x) meets the x-axis.

Hold this question: *Why would a point on the x-axis have anything to do with a zero?*



y = p(x) = x² − 2x − 3

x	p(x)	Point
−2	**5**	(−2, 5)
−1	**0**	(−1, 0)
0	**−3**	(0, −3)
1	**−4**	(1, −4)
2	**−3**	(2, −3)
3	**0**	(3, 0)
4	**5**	(4, 5)

Orange rows: p(x) = 0, so the point is on the x-axis. Each p(x) was found by substituting, for example p(3) = 9 − 6 − 3 = 0.

◆ UNDERSTAND

The graph of y = p(x) is made of the points (x, p(x)). A point is on the x-axis when its y-value is 0. At such a point p(x) = 0, so its **x-coordinate is a zero**. The graph meets the x-axis at (−1, 0) and (3, 0), so the zeroes are **−1 and 3**. Algebra agrees: $x^2 - 2x - 3 = (x - 3)(x + 1)$.

? WHY THIS STEP?

The x-axis is the line y = 0. A point on both the curve y = p(x) and the line y = 0 must satisfy p(x) = 0. That is why the graph meets the x-axis exactly at the zeroes.

★ SPOT THE CLUE

A zero is the **first number** (x-coordinate) of a point on the x-axis. The point (0, −3), where the graph meets the *y*-axis, shows p(0) = −3. That is a value, not a zero.

⚠ COMMON TRAP

Trusting a rough sketch for exact zeroes. A sketch can suggest where zeroes are. Here we are sure about −1 and 3 because p(−1) = 0 and p(3) = 0 were checked by substitution, and the axes are labelled.

✓ CHECK YOURSELF

C5. (a) Which point gives p(0), and what is p(0)? (b) The lowest point of the graph has y = −4. Why is −4 not a zero?

CORE

Sum and product of zeroes

Learning objective: Find the sum and product of the actual zeroes and compare them with the coefficients a, b and c.

Hold this question: *What links the zeroes 2 and 4 of $x^2 - 6x + 8$ to the numbers 1, −6 and 8?*

ZEROES AND COEFFICIENTS: ONE EXAMPLE

$$2x^2 - 5x + 2$$

a = 2 b = −5 c = 2

zeroes: ½ and 2

since $2x^2 - 5x + 2 = (2x - 1)(x - 2)$

SUM

½ + 2 = 5/2 and −b/a = −(−5)/2 = 5/2 ✓

PRODUCT

½ × 2 = 1 and c/a = 2/2 = 1 ✓

Same numbers, found in two completely different ways.

Polynomial	a, b, c	Zeroes	Sum	−b/a	Product	c/a
$x^2 - 6x + 8$	1, −6, 8	2, 4	6	6	8	8
$2x^2 - 5x + 2$	2, −5, 2	½, 2	5/2	5/2	1	1
$3x^2 + 10x - 8$	3, 10, −8	2/3, −4	−10/3	−10/3	−8/3	−8/3

◆ UNDERSTAND

For $ax^2 + bx + c$ with zeroes α and β: **sum α + β = −b/a** and **product αβ = c/a**. In words: sum = −(coefficient of x) ÷ (coefficient of x^2); product = (constant term) ÷ (coefficient of x^2). Page 16 explains *why*.

➤ CHOOSE THE METHOD

To check: (1) write a, b, c with their signs; (2) find the zeroes; (3) add and multiply them; (4) work out −b/a and c/a; (5) compare and write a conclusion.

⚠ COMMON TRAP

Sign check. In the last row b = +10, but the sum is **−10/3**. The minus sign in −b/a is real: write −b first, then divide by a.

✓ CHECK YOURSELF

C13. Find the zeroes of $5x^2 + 7x - 6$. Add and multiply them, then compare with −b/a and c/a.

✎ YOUR TURN

T5 (transfer). For $4x^2 - 4x - 15$, say what the sum and product of the zeroes *must* be *before* you factorise. Then factorise to confirm.

Doubts 1 and 2: what a zero is

DOUBT 1 *"My teacher says the zero of $x - 3$ is 3. But zero should mean 0! What does "zero" mean here?"*

✗ THINKING A zero of a polynomial is the number 0.

◆ EXPLANATION "A zero of p(x)" is any number k that makes the **value** p(k) equal to 0. The word describes the output, not the number k itself.

▦ SEE IT

x	−1	0	1	2	3	4
p(x) = x − 3	−4	−3	−2	−1	0	1

➤ REASONING Ask: which input gives output 0? That input is the zero.

★ MINI-EXAMPLE $p(x) = x - 3$. $p(3) = 0$, so 3 is the zero. $p(0) = -3$, which is not 0, so 0 is *not* a zero of this polynomial.

✎ YOUR TURN DB1 **DB1** Find the zero of $5x - 20$. Is 0 a zero of it? Give a reason.

✓ REMEMBER A zero is an input that gives output 0.

DOUBT 2 *"Why do I write p(x) = 0 to find zeroes? Why not some other equation?"*

✗ THINKING Writing p(x) = 0 is just a rule to memorise.

◆ EXPLANATION By definition a zero is a number where the value is 0, so finding zeroes means solving p(x) = 0. Setting p(x) equal to another number answers a different question.

▦ SEE IT

x	−4	−3	0	3	4
$p(x) = x^2 - 9$	7	0	−9	0	7

➤ REASONING Match the equation to the question: "value is 0" means p(x) = 0.

★ MINI-EXAMPLE $p(x) = x^2 - 9$. $p(x) = 0$ gives $x^2 = 9$, so $x = \pm 3$: these are the zeroes. $p(x) = 7$ gives $x^2 = 16$, so $x = \pm 4$, but $p(4) = 7$, not 0: so 4 is *not* a zero.

✎ YOUR TURN DB2 **DB2** For $p(x) = x^2 - 9$, solve p(x) = 0 and p(x) = 16. Which equation gives the zeroes?

✓ REMEMBER Zeroes come from p(x) = 0, nothing else.

Decision tree: from question to first step

Learning objective: Follow the branches to choose a method, and know which check to run before you finish.

What does the question give or ask?

- **A graph is given**
  → count and read the x-coordinates of points on the x-axis (pp. 11–12)
- **A number is given: is it a zero? Or find k from a zero**
  → substitute: p(k) = 0 (p. 13)
- **The zeroes are asked**
  → factorise by pattern, set each factor to 0 (p. 14)
- **Sum, product, α² + β², or the zeroes are described**
  → write a, b, c with signs; use −b/a and c/a (pp. 15–17, 19)
- **Build a quadratic from zeroes or from S and P**
  → k[x² − Sx + P] (p. 18)
- **Verify the relationship**
  → find the zeroes, then compare with −b/a and c/a (p. 15)

➤ CHOOSE THE METHOD

Before you finish, always check: substitute each zero you found back into p(x); re-do the sign of every −b/a; make sure the final line answers the exact question asked.

⚠ COMMON TRAP

A decision tree is a guide, not a machine. A question can mix ideas: “If the sum of the zeroes of $kx^2 - 12x + 5$ is 6, find k” needs the relationship (sum = 12/k = 6, so k = 2) and then a check that real zeroes exist.

? WHY THIS STEP?

Every branch ends in an action you already know. Choosing the branch is the real skill, which is why page 32 asks you to name the method *before* solving.

✓ CHECK YOURSELF

Try the tree now. Which branch would you take for “Find the zeroes of $x^2 - 3x$ and verify the relationship”? (More than one branch may be needed.)

Mistake clinic (1 of 3)

MISTAKE 1 — A value is not a zero

✗ STUDENT'S WORK	Is 4 a zero of $p(x) = x^2 - 3x - 2$? "$p(4) = 16 - 12 - 2 = 2$, so 2 is the zero."
⚠ WHERE IT FAILS	2 is the *value* $p(4)$. Because the value is not 0, 4 is not a zero. (2 is not a zero either: $p(2) = -4$.)
? WHY TEMPTING	The number at the end of the calculation feels like "the answer".
✓ CORRECTED	$p(4) = 16 - 12 - 2 = 2 \neq 0$, so 4 is **not** a zero of p.
➤ PREVENTION RULE	Only the output 0 means "zero". Any other output is just a value.
✎ FIX IT E1	**E1** A student finds $p(1) = -2$ for $p(x) = x^2 - 4x + 1$ and writes "so -2 is a zero". Correct it.

MISTAKE 2 — A negative input without brackets

✗ STUDENT'S WORK	Find $p(-2)$ for $p(x) = x^2 - 3x + 1$. "$p(-2) = -2^2 - 3(-2) + 1 = -4 + 6 + 1 = 3$"
⚠ WHERE IT FAILS	-2^2 means $-(2 \times 2) = -4$, but the input was -2, so the first term is $(-2)^2 = +4$.
? WHY TEMPTING	Writing the minus sign next to the 2 looks the same as the sign of the number.
✓ CORRECTED	$p(-2) = (-2)^2 - 3(-2) + 1 = 4 + 6 + 1 = 11$.
➤ PREVENTION RULE	Put every negative input in brackets before you do anything else.
✎ FIX IT E2	**E2** A student writes $p(-3)$ for $p(x) = x^2 + 2x - 4$ as $-3^2 + 2(-3) - 4 = -19$. Correct it.

MISTAKE 3 — A rough sketch treated as exact

✗ STUDENT'S WORK	A sketch of $y = x^2 - 2x - 1$ seems to meet the axis near 2.4 and -0.4. "So the zeroes are exactly 2.4 and -0.4."
⚠ WHERE IT FAILS	$p(2.4) = 5.76 - 4.8 - 1 = -0.04 \neq 0$ and $p(-0.4) = 0.16 + 0.8 - 1 = -0.04 \neq 0$. A sketch gives only an approximation.
? WHY TEMPTING	The graph looks as if it passes through 2.4, and a decimal feels exact.
✓ CORRECTED	The sketch only suggests where the zeroes are. The exact zeroes are $1 + \sqrt{2}$ and $1 - \sqrt{2}$ (sum 2 and product -1 agree with $-b/a$ and c/a).
➤ PREVENTION RULE	A zero from a graph needs a labelled point or $p(k) = 0$ by substitution.
✎ FIX IT E3	**E3** A sketch of $y = x^2 - 4x + 1$ seems to meet the x-axis at 0.3 and 3.7. Evaluate $p(0.3)$ and $p(3.7)$. Are these exact zeroes?

Practice 1: foundation

Learning objective: Check the prerequisite ideas and the meaning of a zero.

1 Guided | 2 Part-guided | 3 Independent | 4 Unfamiliar — Answers: Section H.

P1 Write the degree and the name (linear, quadratic or cubic) of each polynomial, and list all its coefficients: (a) $7 - 2x$ (b) $x^3 + 4x$ (c) $3 - x + 5x^2$.

Purpose: Read the anatomy of a polynomial. **Foundation** ★☆☆

Working space (use separate paper if needed)

P2 For $p(x) = 4x^2 + 3x - 5$ find $p(-2)$ and $p(\tfrac{1}{2})$.

Purpose: Substitute with brackets. **Foundation** ★☆☆

Working space (use separate paper if needed)

P3 Is 3 a zero of $x^2 - 2x - 3$? Is -3? Is 1? Give a reason for each.

Purpose: Test zeroes by substitution. **Foundation** ★☆☆

Working space (use separate paper if needed)

P4 Find the zero of each linear polynomial: (a) $6x - 15$ (b) $4 - 3x$ (c) $x/2 + 3$.

Purpose: Solve $ax + b = 0$. **Foundation** ★☆☆

Working space (use separate paper if needed)

P5 In your own words, explain the difference between “the value of p at 3” and “a zero of p”.

Purpose: Separate a value from a zero. **Standard** ★★☆

Working space (use separate paper if needed)

P6 True or false, with a reason: (a) A polynomial of degree 2 has exactly two zeroes. (b) A zero of a polynomial can be the number 0. (c) Every polynomial has at least one zero.

Purpose: Spot over-generalisations. **Standard** ★★☆

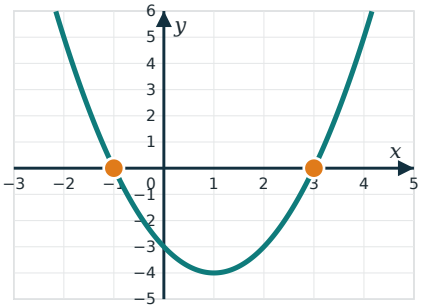
Working space (use separate paper if needed)

Revision map

Learning objective: Recall every key statement, the method clues and the error checklist on one page.

◆ UNDERSTAND

Zero. k is a zero of p(x) if p(k) = 0. Zeroes are the x-coordinates of the points where y = p(x) meets the x-axis (touching counts). A polynomial of degree n has at most n zeroes.
Quadratic $ax^2 + bx + c$ ($a \neq 0$), zeroes α, β: $\alpha + \beta = -b/a$, $\alpha\beta = c/a$. Backwards: $k[x^2 - (\alpha + \beta)x + \alpha\beta]$.
Why: $p(x) = a(x - \alpha)(x - \beta)$ expands to give $b = -a(\alpha + \beta)$ and $c = a\alpha\beta$.



$y = p(x) = x^2 - 2x - 3$

GRAPH ↔ ALGEBRA, ONE PICTURE

zeroes −1 and 3
= x-coordinates where the graph meets the x-axis

sum −1 + 3 = 2
= −b/a = −(−2)/1

product (−1)(3) = −3
= c/a = −3/1

(0, −3) on the y-axis
= the value p(0), not a zero

Question	Method
Is k a zero?	substitute k: p(k) = 0?
Zeroes from a graph	x-coordinates on the x-axis
Find the zeroes	factorise by pattern; solve each factor
Sum, product, $\alpha^2 + \beta^2$	−b/a and c/a, with signs
k from a given zero	put the zero in; set = 0
k from described zeroes	name α, 2α; sum and product
Verify	zeroes first, then compare with −b/a, c/a
Form a quadratic	$k[x^2 - Sx + P]$

⚠ COMMON TRAP

Final error checklist. ☐ a value is not a zero (E1) ☐ bracket negative inputs (E2) ☐ a sketch is not exact (E3) ☐ −b/a has a minus (E4) ☐ c/a, not c (E5) ☐ standard form before a, b, c (E6) ☐ solve each factor = 0 (E7) ☐ common denominators (E8) ☐ compare and conclude (E9)

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