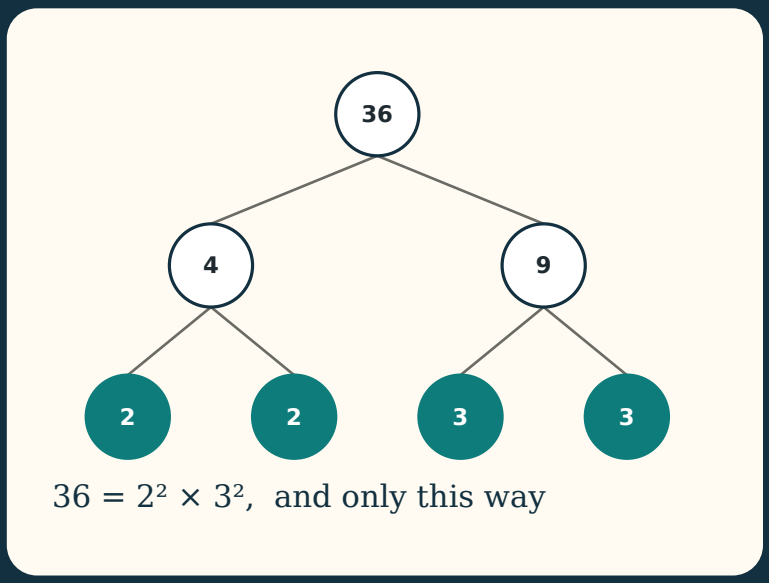
MATHS WITHOUT GUESSWORK

Real Numbers

Understand the Concept.
Recognise the Method.
Solve Independently.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27



$36 = 2^2 \times 3^2$, and only this way

Core topics: the Fundamental Theorem of Arithmetic · Proofs of irrationality

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher.

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✓ WHERE ARE THE ANSWERS?

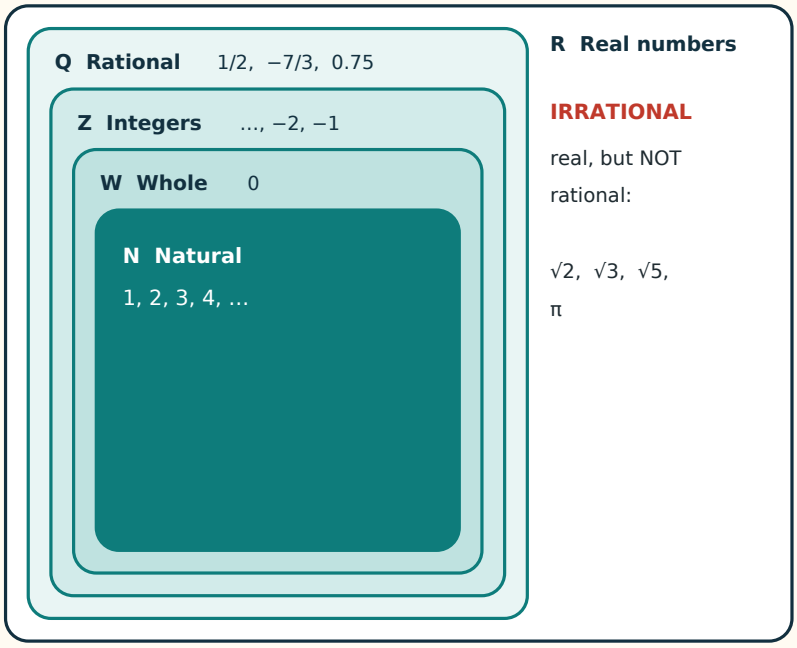
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PREREQUISITE

The number families

Learning objective: Place any number in its families and tell a rational number from an irrational one.

Numbers live in nested families. Each bigger family contains all the smaller ones.



N: counting numbers 1, 2, 3, ...
W: N together with 0
Z: whole numbers and their negatives
Q: numbers that can be written p/q with p, q integers and $q \neq 0$
Irrational: a real number that is not rational
R: all rational and irrational numbers together (every point on the number line)

◆ UNDERSTAND

Every integer is rational: $5 = 5/1$ and $-3 = -3/1$. So a number can sit in several families at once; the *smallest* one tells you the most.

★ SPOT THE CLUE

A root sign does **not** decide it. $\sqrt{9}$ looks irrational but equals 3, so it is a natural number. Always simplify first, then classify.

⚠ COMMON TRAP

“Every square root is irrational.” False: $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{16} = 4$. In Section C you will prove that $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ are irrational. Those are results to be proved, not guesses.

✓ CHECK YOURSELF

C1. Name the *smallest* family (N, W, Z, Q or irrational) for each: -3, 0, $7/2$, $\sqrt{9}$, $\sqrt{7}$, $10/5$. Give a one-line reason for $\sqrt{9}$ and $10/5$.

Why does this matter? A proof that a number is irrational only makes sense once you know exactly what “rational” means. Every such proof starts from the form p/q.

CORE

The Fundamental Theorem of Arithmetic

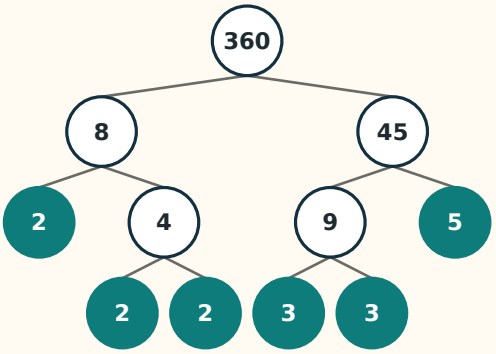
Learning objective: State the theorem, explain what “unique” means, and test it on two different factor trees.

Hold this question: *If I split 360 in different ways, could I ever end up with different primes?*

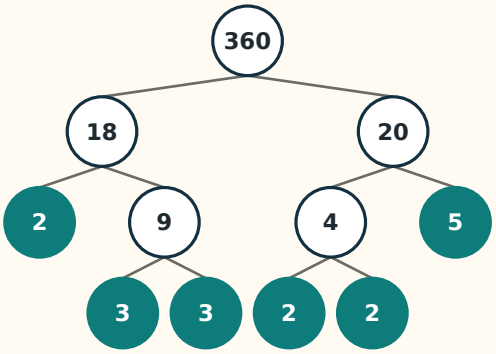
◆ UNDERSTAND

Fundamental Theorem of Arithmetic (FTA). Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
Existence: a prime factorisation always exists. **Uniqueness:** there is only one list of primes, with fixed counts, that multiplies to the number.

Tree A: 360 = 8 × 45 …



Tree B: 360 = 18 × 20 …



Tree A leaves: 2, 2, 2, 3, 3, 5. Tree B leaves: 2, 3, 3, 2, 2, 5. Both give $360 = 2^3 \times 3^2 \times 5$. Check: $8 \times 9 \times 5 = 72 \times 5 = 360$ ✓

? WHY THIS STEP?

The two trees look different, yet the leaves are the same primes in the same amounts. That is what “unique” means. Think of the factorisation as the number’s **fingerprint**: two numbers are equal exactly when their prime fingerprints match.

⚠ COMMON TRAP

“Different trees mean different factorisations.” No: a tree is only a *method*. The FTA is about the primes at the ends, and those never change.

✓ CHECK YOURSELF

C7. Factorise 300 using two different first splits (for example 3 × 100 and 30 × 10). Confirm that both give the same prime factorisation.

✎ YOUR TURN

T1 (transfer). Priya says: “36 = 2 × 18 and 36 = 3 × 12, so 36 has two different factorisations and the FTA is false.” Is she right? Explain.

CORE

Proof that √2 is irrational

Learning objective: Write and explain a complete proof that √2 is irrational, naming the reason for every line.

Hold this question: *Where exactly does the contradiction come from, and why is it a real contradiction?*

➤ CHOOSE THE METHOD

To prove: $\sqrt{2}$ is irrational. Because we must rule out every fraction, we use proof by contradiction, with the Key Fact (p. 15) as the engine.

#	Statement	Reason
1	Assume $\sqrt{2}$ is rational.	We want a contradiction, so assume the opposite of the claim.
2	Then $\sqrt{2} = p/q$ with p, q positive integers, $q \neq 0$, and p, q **coprime**.	Definition of rational; any fraction can be reduced to lowest terms (p. 8). $\sqrt{2} > 0$, so p and q can be positive.
3	Squaring: $2 = p^2/q^2$, so $p^2 = 2q^2$.	Squaring keeps equality; multiply both sides by q^2.
4	So 2 divides p^2.	$p^2 = 2 \times$ (the integer q^2).
5	So 2 divides p.	Key Fact with prime 2 (p. 15).
6	Write $p = 2m$ for some integer m.	Meaning of "2 divides p".
7	Then $(2m)^2 = 2q^2$, so $4m^2 = 2q^2$, so $q^2 = 2m^2$.	Substitute and divide by 2.
8	So 2 divides q^2, hence 2 divides q.	Same reasoning as rows 4 and 5.
9	Now 2 divides p **and** 2 divides q.	Rows 5 and 8.
10	This contradicts "p and q are coprime".	Coprime means 1 is the only common factor; 2 is a common factor greater than 1.
11	So the assumption was false: $\sqrt{2}$ is irrational. ∎	Every other step was valid, so only row 1 can be wrong.

? WHY THIS STEP?

Why is row 10 a true contradiction? Row 2 promised that p and q have *no* common factor above 1. Rows 3 to 9 then produced one (the number 2). The proof defeats its own assumption, not some outside fact.

✓ CHECK YOURSELF

C13. (a) Why are we allowed to assume p and q are coprime in row 2? (b) Which two rows together create the contradiction?

Doubts 1 and 2: theorems and primes

DOUBT 1 *"My teacher says "by the theorem…", but what does a theorem actually tell me? Is it just a rule I have to memorise?"*

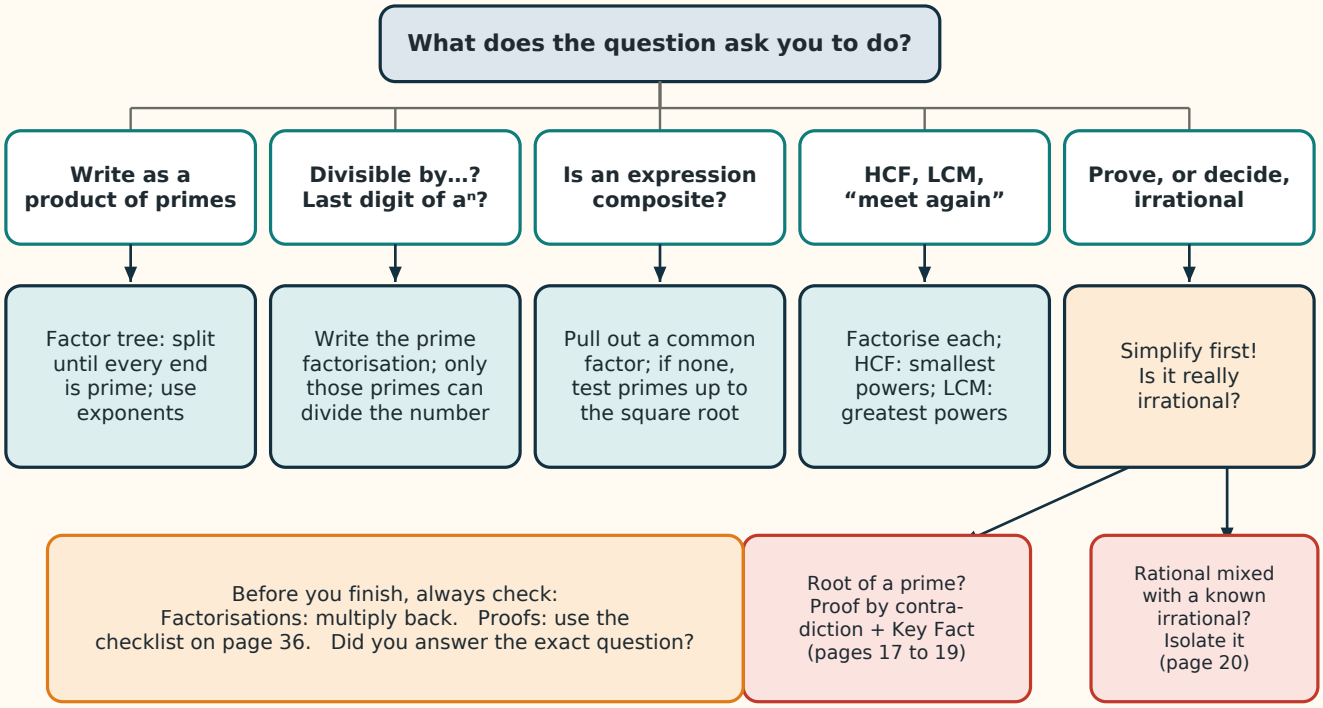
✗ THINKING	A theorem is a rule from the book. I only need to remember the words.
◆ EXPLANATION	A theorem is a statement that has been **proved** from facts already accepted. It has two parts: a condition (the "if") and a conclusion (the "then"). It promises the conclusion *only when the condition holds*.
➤ REASONING	Before using any theorem ask: (1) What must be true first? (2) What does it then promise? If (1) fails, the theorem says nothing.
★ MINI-EXAMPLE	**Key Fact:** *if* p is prime and p divides a^2, *then* p divides a. With p = 3 and $a^2 = 36$ the condition holds (3 divides 36), so 3 divides a = 6 ✓. With p = 4, "p is prime" fails, so nothing is promised; indeed 4 divides 36 but not 6.
✎ YOUR TURN DB1	Write the "if" part and the "then" part of the Key Fact. Then decide: is "if p divides a, then p divides a^2" the same statement? Is it true?
✓ REMEMBER	A theorem is a condition plus a conclusion. Check the condition first.

DOUBT 2 *"Why the fuss about prime factorisation? I can split 12 as 3 × 4 or as 2 × 6. Why must the factors be primes?"*

✗ THINKING	Any split into factors is as good as any other, so the answer should not matter.
◆ EXPLANATION	Splits into *any* factors are not unique: 12 = 3 × 4 = 2 × 6 = 1 × 12. Primes cannot be split further, and the FTA says the primes you end with are always the same. So the prime factorisation is a fixed fingerprint of the number.
➤ REASONING	To compare numbers or decide what divides them, you need something that cannot change. Only the prime factorisation has that property.
★ MINI-EXAMPLE	12 = 3 × 4 and 12 = 2 × 6 look different. Split fully: 3 × (2 × 2) and 2 × (2 × 3). Both become $2^2 \times 3$. Check: 4 × 3 = 12 ✓.
✎ YOUR TURN DB2	Write 30 as a product of two numbers in three different ways. Then give its prime factorisation.
✓ REMEMBER	Many factorisations, but only one prime factorisation.

Decision tree: from question to first step

Learning objective: Follow the branches to choose a method, and know which check to run before you finish.



⚠ COMMON TRAP

A decision tree is a guide, not a machine. A question can mix ideas: for example “show that 6^n cannot end in 0” starts as an **a^n question** but is really about which primes are present. Always inspect the structure.

? WHY THIS STEP?

Every branch ends in an action you already know. Choosing the branch is the real skill: that is why page 32 asks you to name the method *before* solving.

✓ CHECK YOURSELF

Try the tree now. Which branch would you take for “Is √16 irrational?” (Hint: look at the bottom-right box first.)

Mistake clinic (1 of 3)

MISTAKE 1 · Evidence is not proof

✗ STUDENT'S WORK	*Prove that √3 is irrational.* "√3 = 1.7320508... The digits never end and never seem to repeat, so √3 is irrational."
⚠ WHERE IT FAILS	Nothing shows that **no** fraction p/q equals √3. Only finitely many digits can ever be seen.
? WHY TEMPTING	A long, patternless decimal *feels* convincing, and the calculator makes it look like data.
✓ CORRECTED	Assume √3 = p/q with p, q coprime integers, q ≠ 0. Then $p^2 = 3q^2$, so 3 divides p, and so on, ending with 3 dividing both p and q, which contradicts "coprime" (page 18).
➤ PREVENTION RULE	A decimal can suggest a result. Only a logical argument proves it.
✎ FIX IT E1	Explain why "I tried n = 1, 2, 3, 4 and 14^n never ended in 0, so it never does" is not a proof. What argument is needed?

MISTAKE 2 · Stopping before every factor is prime

✗ STUDENT'S WORK	*Find the prime factorisation of 180.* "180 = 2 × 9 × 10."
⚠ WHERE IT FAILS	9 = 3 × 3 and 10 = 2 × 5 are composite. The product equals 180, but the factorisation is unfinished.
? WHY TEMPTING	The multiplication checks (2 × 9 × 10 = 180), so it feels finished.
✓ CORRECTED	180 = 2 × (3 × 3) × (2 × 5) = $2^2 \times 3^2 \times 5$. Check: 4 × 9 × 5 = 180 ✓.
➤ PREVENTION RULE	Scan every factor and ask: is it prime?
✎ FIX IT E2	A student writes 144 = 4 × 36 as "the prime factorisation of 144". Correct it and check your answer.

MISTAKE 3 · Using the Key Fact for a non-prime

✗ STUDENT'S WORK	"$36 = 6^2$ is divisible by 4, so 4 divides 6."
⚠ WHERE IT FAILS	The Key Fact needs a **prime** divisor. $4 = 2^2$ is not prime, and 4 does not divide 6.
? WHY TEMPTING	It copies the pattern "p divides a^2, so p divides a" that works for primes such as 2 and 3.
✓ CORRECTED	4 divides 36 (36 = 4 × 9), but 6 = 2 × 3 has only one 2. What is true: the prime 2 divides 36, so 2 divides 6.
➤ PREVENTION RULE	Before using the Key Fact, check the word "prime".
✎ FIX IT E3	A student says: "9 divides 12^2, so 9 divides 12." Is that right? Which true statement about 3 and 12 can you make instead?

Practice 1: foundation

Learning objective: Check the prerequisite ideas you will rely on later.

1 Guided | 2 Part-guided | 3 Independent | 4 Unfamiliar

Answers: Section H.

P1 Name the *smallest* family (N, W, Z, Q or irrational) for each number: 0, -5, $3/8$, $\sqrt{25}$, $\sqrt{3}$, $12/4$. (You may use that $\sqrt{3}$ is irrational, page 18.)

Purpose: Classify numbers; simplify before deciding.

easy ★☆☆

Working space (use separate paper if needed)

P2 Decide whether each number is prime or composite, with a reason: **57, 61, 87, 101**.

Purpose: Test primality using primes up to the square root.

easy ★☆☆

Working space (use separate paper if needed)

P3 Find the prime factorisation of 420 using a factor tree. Check by multiplying back.

Purpose: Complete a factorisation and verify it.

easy ★☆☆

Working space (use separate paper if needed)

P4 Write $84/126$ in lowest terms. Show the common factor you divided by.

Purpose: Reduce to lowest terms, as every irrationality proof assumes.

easy ★☆☆

Working space (use separate paper if needed)

P5 Are 35 and 48 coprime? Justify using prime factorisation.

Purpose: Apply the definition of coprime.

easy ★☆☆

Working space (use separate paper if needed)

P6 True or false, with a reason: **(a)** Every odd number greater than 1 is prime. **(b)** 2 is the only even prime number.

Purpose: Spot over-generalisations.

moderate ★★☆

Working space (use separate paper if needed)

Revision map

Learning objective: Recall every key statement, the proof structure and the final error checklist on one page.

◆ UNDERSTAND

FTA. Every composite number can be written as a product of primes, and this factorisation is unique apart from the order of the factors.
Key Fact. If a prime p divides a^2, then p divides a (it needs p to be prime).
Rational: can be written p/q, p, q integers, $q \neq 0$. **Coprime:** only common positive factor is 1.
Irrational: real but not rational.



➤ CHOOSE THE METHOD

Pattern 1: √n for a prime n. Assume $\sqrt{n} = p/q$ (coprime). Then $p^2 = nq^2$, so n divides p^2, so n divides p (Key Fact). Write $p = nm$. Then $q^2 = nm^2$, so n divides q. Common factor n contradicts coprime.
Pattern 2: rational combined with a known irrational. Assume the number equals a/b. Isolate the known irrational. It becomes a ratio of integers: rational. This contradicts what is known.

Method quick map

Write as a product of primes	Factor tree, until every end is prime, then multiply back
Divisible by? Last digit of a^n?	Prime factorisation of a^n, then uniqueness: only those primes
Is an expression composite?	Pull out a common factor; otherwise test primes up to the square root
HCF / LCM (enrichment)	Smallest powers for HCF, greatest powers for LCM
Prove √prime is irrational	Pattern 1
Prove a + b√n irrational	Pattern 2

⚠ COMMON TRAP

Final error checklist. ☐ Evidence is not proof (E1) ☐ Every factor prime (E2) ☐ Key Fact only for primes (E3) ☐ A reason for every "so" (E4) ☐ Symbols defined (E5) ☐ Coprime stated (E6) ☐ Squared the coefficient too (E7) ☐ Contradiction named (E8) ☐ Conclusion written, exact question answered (E9)

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- **Foundation and core methods.** 16 pages of concept explanations, worked examples and proofs, with "Why this step?" notes.
- **Concept checks and transfer questions.** 16 quick checks (C1–C16) and 8 transfer questions (T1–T8) in slightly new forms.
- **Doubt Bank.** 16 common doubts, each with the faulty thinking, a clear explanation and a "Your turn" question.
- **Method recognition.** A clue table, a decision tree, 5 look-alike question pairs and 8 "choose the method first" questions.
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