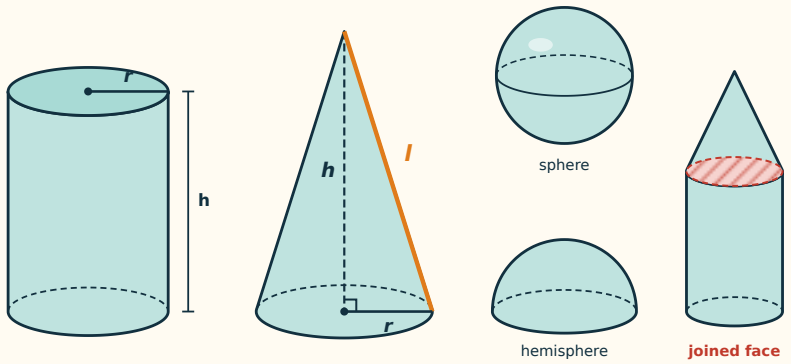
MATHS WITHOUT GUESSWORK

Surface Areas and Volumes

Identify the Solid.
Choose the Formula.
Solve Independently.

A guided problem-solving workbook

CBSE Class 10 Mathematics · Syllabus session 2026–27



Knowing a formula is not enough. Choosing the right one is the skill.

Core topics: cylinder · cone · sphere · hemisphere · cuboid · surface areas and volumes of combinations of two solids

Original explanations, examples and questions. Not a substitute for the NCERT textbook or your teacher. Diagrams are not drawn to scale.

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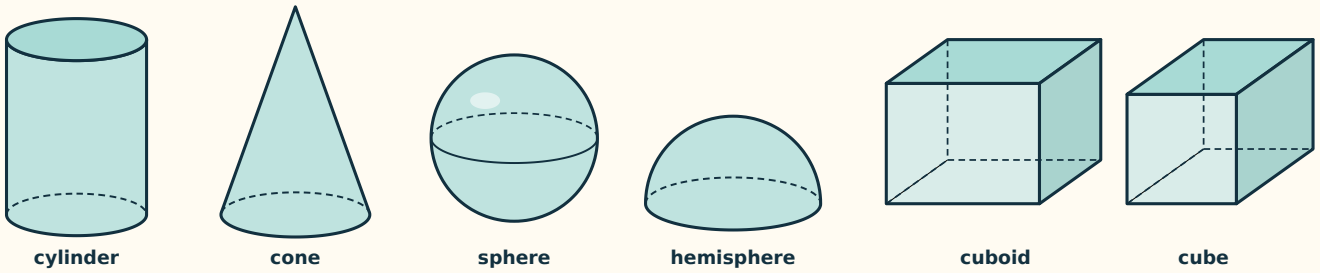
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Flat shapes and solids

PREREQUISITE

Learning objective: Describe each solid by its faces: which are flat, which are curved, and how many there are.



Curved surfaces are pale teal; flat faces are darker teal. Dashed lines are hidden edges at the back.

◆ UNDERSTAND

A **circle** or a **rectangle** is flat (2-D): it has only area. A **solid** (3-D) has length, breadth and height. It has **surfaces**, which have area, and it encloses **space**, which has volume.

Every solid in this chapter is built from two kinds of surface: **flat faces** (circles, rectangles, squares) and **curved surfaces** (the side of a can, the outside of a ball).

Solid	Everyday object	Flat faces	Curved surfaces	Model limits
Cylinder	tin can, pipe, pillar	2 circles (if closed)	1	A real can has a rim and thin walls
Cone	ice-cream cone, tent top	1 circle	1	A real cone may have a rounded tip
Sphere	ball, marble	0	1	A football has seams and air inside
Hemisphere	bowl, dome	1 circle	1	A bowl is hollow and open
Cuboid / cube	box, brick, die	6 rectangles / squares	0	Edges may be rounded

? WHY THIS STEP?

Why count faces? Because surface-area formulas are just sums of face areas. If you know a closed cylinder has one curved surface and two circles, you can *build* its TSA instead of remembering it: $2\pi rh + \pi r^2 + \pi r^2$.

⚠ COMMON TRAP

"A sphere and a circle are the same shape." A circle is flat and has an area πr^2. A sphere is a solid ball: its *surface* is $4\pi r^2$ and it encloses a *volume* $\frac{4}{3}\pi r^3$.

✓ CHECK YOURSELF

C1. How many flat faces and how many curved surfaces does each have? (a) a closed cylinder (b) a cylinder open at the top (c) a pipe open at both ends (d) a solid hemisphere (e) a sphere.

Concept 1: the cylinder (surfaces)

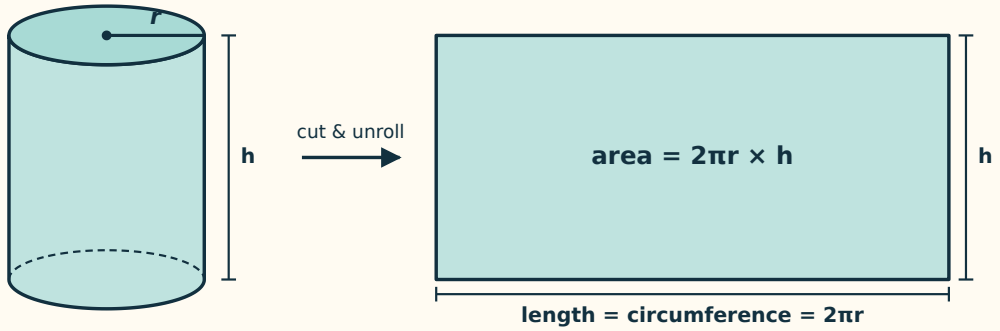
CORE

Learning objective: Explain why CSA = 2πrh, and build the TSA of a closed cylinder from its faces.

Hold this question: *A label wraps exactly once round a tin can. Why does its length equal the circumference of the can?*

◆ UNDERSTAND

Object first. A tin can, a pipe, a pillar. A **right circular cylinder** has two equal circular ends (radius r) joined by a curved surface, with height h at right angles to the ends.



➤ CHOOSE THE FORMULA

CSA = 2πrh **area of one end = πr²** **TSA (closed) = 2πrh + 2πr² = 2πr(r + h)**

WORKED EXAMPLE **Closed cylinder, r = 7 cm, h = 10 cm**

CSA = $2 \times \frac{22}{7} \times 7 \times 10 =$ **440 cm²**

Two ends = $2 \times \frac{22}{7} \times 7 \times 7 = 308$ cm²

TSA = 440 + 308 = **748 cm²** (or 2πr(r + h) = 44 × 17 = 748 ✓)

? WHY THIS STEP?

Why the circumference? The label goes round the can exactly once, so its length is the distance round the circular end: 2πr. Its width is the height h. A rectangle's area is length × width, so CSA = 2πr × h.

★ SPOT THE CLUE

Use CSA for: label, side wall, pillar painted (top and bottom not), pipe open at both ends. **Use TSA** only if both ends are part of the surface asked for.

⚠ COMMON TRAP

2πr² for the ends. That is right only for *two* ends. An open container has **one** end: πr².

✎ YOUR TURN

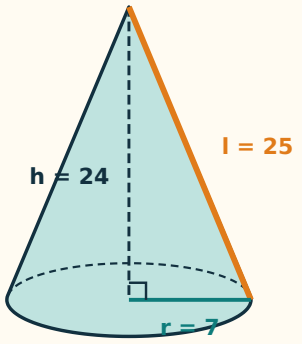
T1. A closed cylindrical tin has radius 3.5 cm and height 10 cm. Find (a) the area of its label (curved surface) and (b) the area of tin sheet needed to make it.

Concept 2: the cone and its two heights

CORE

Learning objective: Identify which height a question gives and which height each cone formula needs.

Hold this question: *A cone has slant height 25 cm and radius 7 cm. Can I put 25 into the volume formula?*



◆ UNDERSTAND

Object first. An ice-cream cone, a birthday cap, the top of a tent. A **right circular cone** has a circular base and an apex directly above the centre of the base.

It has **two heights**: the perpendicular height **h** (inside the cone, centre to apex) and the slant height **l** (on the surface, edge to apex). They are linked by $\mathbf{l^2 = r^2 + h^2}$.

Formula	Needs	Why that height?
CSA = πrl	slant height l	The curved surface lies along the slope; when unrolled it is a sector whose radius is l.
V = ⅓πr²h	perpendicular height h	Volume measures how high the space rises above the base, straight up.

WORKED EXAMPLE **r = 7 cm, h = 24 cm: find l**

$l^2 = r^2 + h^2 = 49 + 576 = 625$

$l =$ **25 cm**

WORKED EXAMPLE **r = 7 cm, l = 25 cm: find h**

$h^2 = l^2 - r^2 = 625 - 49 = 576$

$h =$ **24 cm**

→ PREDICT THE NEXT STEP

A question gives r and l and asks for the **volume**. What must you calculate *before* using ⅓πr²h? (Answer: h, from $h^2 = l^2 - r^2$.)

⚠ COMMON TRAP

Swapping heights. ⅓π × 49 × 25 uses l in the volume formula. The true volume (with h = 24) is 1232 cm³; with 25 you get 1283.33 cm³, which is wrong.

✎ YOUR TURN

T3. A cone has radius 9 cm and slant height 15 cm. (a) Find its perpendicular height. (b) Which of the two heights would you use for its CSA, and which for its volume?

Doubts 1 and 2: area or volume? CSA or TSA?

DOUBT 1 *"How do I know whether the question asks for surface area or volume?"*

✗ WHY THIS CONFUSION HAPPENS	Both come from the same measurements of the same solid, so students react to the solid, not to the question.
◆ UNDERSTAND	Surface area measures **covering** (paint, paper, sheet, canvas). Volume measures **filling or occupying** (water, air, sand, metal, capacity).
★ SPOT THE CLUE	Covering verbs: paint, wrap, polish, cover, canvas, sheet. Filling words: hold, contain, capacity, litres, melt, how much metal.
➤ CHOOSE THE FORMULA	Covering → an area formula (cm^2). Filling → a volume formula (cm^3 or litres).
✎ SOLVED EXAMPLE	A closed cubical gift box has edge 10 cm. Paper to wrap it (no overlap) = 6 × 10 × 10 = **600 cm^2**. Space inside = 10^3 = **1000 cm^3**.
? WHY THIS STEP?	Wrapping touches every face, so all six face areas are added. Filling counts unit cubes, so three edges are multiplied.
⚠ COMMON TRAP	Giving 1000 cm^2 of paper because "it is the bigger number". The unit cm^2 cannot measure space.
✎ YOUR TURN DB1	A closed cylindrical tin has r = 7 cm and h = 10 cm. Find (a) the tin sheet needed to make it and (b) how much it holds.
✓ CHECK YOURSELF	One answer is in cm^2, the other in cm^3. Check: the sheet answer must be more than the CSA alone.

DOUBT 2 *"What is the real difference between CSA and TSA?"*

✗ WHY THIS CONFUSION HAPPENS	Both are "surface area", and for a sphere there is no difference, so students think there never is.
◆ UNDERSTAND	CSA is only the curved part. TSA is every surface: curved **plus** flat faces. TSA − CSA = the flat faces.
★ SPOT THE CLUE	"Curved", "lateral", "side", "label" → CSA. "Total", "whole surface", "closed", "solid … all over" → TSA.
➤ CHOOSE THE FORMULA	Cylinder: CSA $2\pi rh$, TSA $2\pi r(r + h)$. Cone: CSA πrl, TSA $\pi r(l + r)$. Hemisphere: CSA $2\pi r^2$, TSA $3\pi r^2$.
✎ SOLVED EXAMPLE	Closed cylinder, r = 7 cm, h = 10 cm: CSA = 2 × 22/7 × 7 × 10 = **440 cm^2**; TSA = 440 + 2 × 154 = **748 cm^2**.
? WHY THIS STEP?	The two ends are separate circles of area πr^2 = 154 cm^2 each, added only for TSA.
⚠ COMMON TRAP	Using TSA for a label, which would also cover the top and bottom of the tin.
✎ YOUR TURN DB2	A cone has r = 7 cm and slant height 10 cm. Find its CSA and its TSA.
✓ CHECK YOURSELF	TSA − CSA should equal one circle of radius 7 cm.

Question clue → solid → quantity → formula

Learning objective: Translate the words of a question into a solid, a quantity and a first step.

Question clue	Solid	Quantity	Formula	First step
"Canvas for a conical tent"	cone (no floor)	curved surface	πrl	find l if h is given
"Tin to make a closed can"	closed cylinder	TSA	$2\pi r(r + h)$	halve d; check both ends exist
"Water a glass can hold"	cylinder	capacity (volume)	$\pi r^2 h$	use inner r; convert to litres
"Paint the outside of a bowl"	hemisphere (open)	outer CSA	$2\pi r^2$	no flat face: it is open
"Polish a solid hemisphere all over"	solid hemisphere	TSA	$3\pi r^2$	include the flat face
"Leather to make a football"	sphere	surface area	$4\pi r^2$	halve d
"Metal in a solid ball"	sphere	volume	$\frac{4}{3}\pi r^3$	r cubed; cm^3
"Sand in a conical heap"	cone	volume	$\frac{1}{3}\pi r^2 h$	need h, not l
"Paint a toy: cone on a hemisphere"	combination	exposed surface	$\pi rl + 2\pi r^2$	joined circle not painted
"Air inside a tent: cylinder + cone"	combination	total volume	$\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$	add the two volumes
"Wood left after a cone is scooped out of a cylinder"	combination (removed)	volume	$\pi r^2 h - \frac{1}{3}\pi r^2 h$	subtract the removed part
"Ribbon round the rim of a can"	circle (edge)	a length	$2\pi r$	not an area at all

➤ WHY THIS FORMULA?

Read the table from left to right, never right to left. The formula is the *result* of the three columns before it. Two rows can share a solid (sphere: leather vs metal) and still need different formulas, because the quantity is different.

⚠ COMMON TRAP

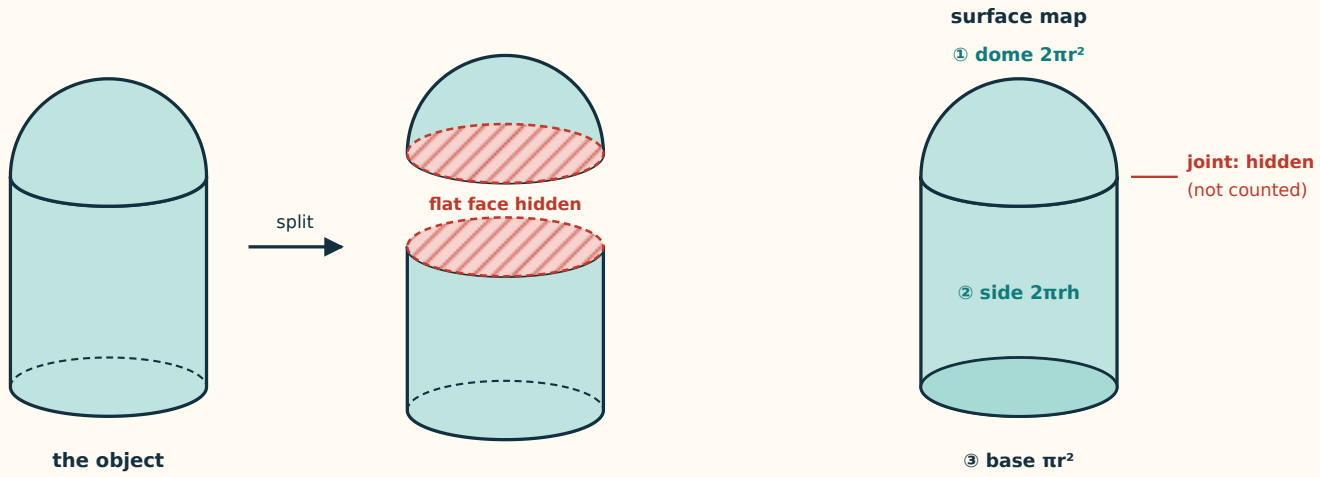
Same clue, different object. "Paint the dome" (open, CSA $2\pi r^2$) and "paint the paperweight" (solid, TSA $3\pi r^2$) both say "paint" and both are hemispheres. The deciding column is the third: which surfaces exist. That is why the formula comes last.

✎ EXPLAIN IT IN YOUR OWN WORDS

Pick any two rows that share a solid. In your own words, explain why their formulas differ.

Combined solids: volumes add, surfaces do not

Learning objective: Split an object into two solids, then treat its volume and its surface differently.



◆ UNDERSTAND

The critical rule. Volume: if the two parts do not overlap, volume of object = volume of part 1 + volume of part 2. (If a part is *removed*, subtract it.)
Surface: count only the surfaces on the **outside** of the finished object. A face that is glued to the other part is inside the object and is not counted.

WORKED EXAMPLE **Three different operations, one object (cylinder r = 7 cm, h = 10 cm, hemisphere on top)**

Operation	Working	Result	Is it the answer?
Add the volumes	1540 + 718.67	2258.67 cm³	✓ volume of the object
Add the two TSAs	748 + 462	1210 cm²	✗ counts the joint twice
Add exposed surfaces	308 + 440 + 154	902 cm²	✓ surface of the object

Check: 1210 − 902 = 308 = 2 × 154, exactly two copies of the hidden circle.

The nine-step method for any combined solid

1	Observe the whole object.	4	List given dimensions (and find hidden ones: hemisphere height = r).	7	Choose a formula for each part.
2	Name the two component solids.	5	Identify the quantity asked.	8	Add or subtract logically.
3	Sketch the parts separately.	6	Volume? Add parts. Surface? Draw a surface map.	9	Check units and size.

⚠ COMMON TRAP

"Volumes add, so surfaces add." Volumes are the space inside; joining keeps all of it. Surfaces are the skin outside; joining buries part of it.

Mistake clinic (1 of 3): measurements

MISTAKE 1 · Diameter used as radius

✗ STUDENT'S WORK	*Volume of a sphere of diameter 14 cm.* "⁴⁄₃ × 22/7 × 14³ = 11 498.67 cm³"
⚠ WHY IT IS WRONG	The formula needs r = 7 cm. Cubing the diameter makes the volume 8 times too big.
✓ CORRECTED	r = 7 cm; V = ⁴⁄₃ × 22/7 × 343 = 1437.33 cm³.
➤ QUICK RULE	Write "r = d ÷ 2 = ..." first.
✎ FIX IT E1	Find the surface area of a sphere of diameter 21 cm.

MISTAKE 2 · CSA given when TSA is needed

✗ STUDENT'S WORK	*Tin to make a closed can, r = 7 cm, h = 10 cm.* "2πrh = 440 cm²"
⚠ WHY IT IS WRONG	A closed can also has a top and a bottom, made of tin too.
✓ CORRECTED	TSA = 2πr(r + h) = 44 × 17 = 748 cm².
➤ QUICK RULE	"Make a closed..." → every face.
✎ FIX IT E2	Find the TSA of a closed cylinder with r = 14 cm and h = 6 cm.

MISTAKE 3 · Slant height in the cone's volume

✗ STUDENT'S WORK	*Cone r = 6 cm, l = 10 cm; volume.* "⅓ × π × 36 × 10 = 120π cm³"
⚠ WHY IT IS WRONG	Volume needs the perpendicular height h, not l.
✓ CORRECTED	h = √(100 − 36) = 8 cm; V = ⅓π × 36 × 8 = 96π cm³.
➤ QUICK RULE	Volume → h. Surface → l.
✎ FIX IT E3	A cone has r = 9 cm and l = 15 cm. Find its volume in terms of π.

MISTAKE 4 · Perpendicular height in the cone's CSA

✗ STUDENT'S WORK	*Cone r = 3 cm, h = 4 cm; CSA.* "π × 3 × 4 = 12π cm²"
⚠ WHY IT IS WRONG	The curved surface lies along the slope, so it needs l = √(9 + 16) = 5 cm.
✓ CORRECTED	CSA = π × 3 × 5 = 15π cm².
➤ QUICK RULE	Before πrl, check you have l.
✎ FIX IT E4	Find the CSA of a cone with r = 5 cm and h = 12 cm, in terms of π.

Practice 1: recognise

Learning objective: Recognise solids, quantities, measurements and units before calculating anything.

1 Recognise | 2 Apply | 3 Connect | 4 Board-style *Mostly no calculation. Answers: Section J.*

P1 The unit of the curved surface area of a cone, when lengths are in cm, is:

(a) cm (b) cm² (c) cm³ (d) cm⁴

Purpose: Units match quantities. **easy ★☆☆**

Working space (use separate paper if needed)

P2 The volume of a hemisphere of radius r is:

(a) $2\pi r^2$ (b) $3\pi r^2$ (c) $\frac{2}{3}\pi r^3$ (d) $\frac{4}{3}\pi r^3$

Purpose: Tell surface formulas from volume formulas. **easy ★☆☆**

Working space (use separate paper if needed)

P3 Name the two solids that model each object: (a) a circus tent (b) a medicine capsule (c) an ice-cream cone with a scoop on top (d) a sharpened pencil (e) a test tube.

Purpose: Decompose real objects. **easy ★☆☆**

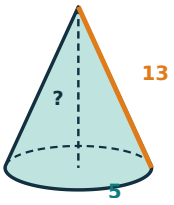
Working space (use separate paper if needed)

P4 CSA, TSA, volume or another surface? (a) paint the outside of a closed water tank (b) the water the tank can hold (c) a label round a can (d) canvas for a tent with no floor (e) silver to coat a ball.

Purpose: Decide what is asked. **moderate ★★☆**

Working space (use separate paper if needed)

P5 The cone in the figure has radius 5 cm and slant height 13 cm. (a) Find the perpendicular height. (b) Which height goes into the CSA formula, and which into the volume formula?



Purpose: Two heights of a cone. **moderate ★★☆**

Working space (use separate paper if needed)

P6 Two cubes of edge 3 cm are joined face to face. The surface area of the new solid is:

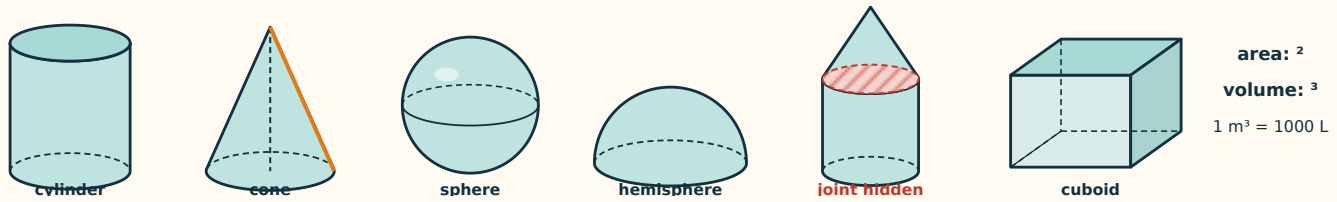
(a) 108 cm² (b) 90 cm² (c) 72 cm² (d) 54 cm²

Purpose: Hidden faces. **moderate ★★☆**

Working space (use separate paper if needed)

Revision sheet

Learning objective: Recall every prescribed formula with its condition, on one page.



Solid	Curved / lateral	Total surface	Volume	Remember
Cuboid l, b, h	2(l + b)h	2(lb + bh + hl)	lbh	Open box: lb + 2(l + b)h
Cube a	$4a^2$	$6a^2$	a^3	Joined cubes hide faces
Cylinder r, h	2πrh	2πr(r + h)	$\pi r^2 h$	Open top: 2πrh + πr²; pipe: 2πrh
Cone r, h, l	πrl	πr(l + r)	$\frac{1}{3}\pi r^2 h$	$l^2 = r^2 + h^2$; area → l, volume → h
Sphere r		$4\pi r^2$	$\frac{4}{3}\pi r^3$	No flat face
Hemisphere r	$2\pi r^2$	$3\pi r^2$ (solid)	$\frac{2}{3}\pi r^3$	Bowl → $2\pi r^2$; height = r

⚖ KNOWING ≠ CHOOSING

Knowing says: the formula table above

1 · WHICH SOLID?	Name it (or the two solids).
2 · WHAT IS ASKED?	Covering → area. Filling, holding, metal → volume. Rim, ribbon → length.
3 · WHICH SURFACES?	Open? Joined? Removed part? Build the surface from the faces that exist.
4 · WHICH MEASUREMENTS?	r (not d); for a cone, l or h as the formula needs; one unit for everything.

SO CHOOSE **Only now write the formula, then substitute.**

◐ INSIDE OR OUTSIDE?

Combined solids. Volume: add the parts (subtract any removed part). Surface: only outside faces. Joined circle → not counted. Smaller solid on a bigger face → $-\pi r^2$ (covered) + its exposed surface. Scooped out → new inner surface added.

◆ RULES

Recasting ENRICHMENT: volume before = volume after (× number of pieces), assuming no loss. Never equate surface areas.
Units: 1 m = 100 cm; 1 m^2 = 10 000 cm^2; 1 m^3 = 1 000 000 cm^3 = 1000 L; 1 L = 1000 cm^3.

Object in the question	Model	Usually asked	Watch for
tin, drum, pillar, pipe, well	cylinder	TSA / CSA / volume	open ends; well → volume of earth
tent, rocket, pencil, silo	cylinder + cone	canvas / paint; air or grain	no floor; find l
capsule, test tube, gulab jamun	cylinder + hemisphere(s)	surface; volume	cylinder length = total − r (or − 2r)
toy top, ice-cream with a scoop; bowl, dome	cone + hemisphere; open hemisphere	paint; volume; capacity	cone height = total − r; a bowl has no flat face

⚠ COMMON TRAP

Top mistakes: d used as r · l in a cone's volume · h in πrl · bowl given $3\pi r^2$ · closed TSA for open objects · joints counted · TSAs added · units mixed · ² for volume · wrong π · early rounding · formula with no reason.

MATHS WITHOUT GUESSWORK

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- **Foundation.** Concept explanations that build the ideas the chapter depends on.
- **Core formulas.** Worked examples with a reason for every step, and quick checks to try.
- **Doubt Bank.** 16 common doubts answered clearly, each followed by a question to try.
- **Formula selection.** Clue tables, decision charts and look-alike questions that train you to choose the method.
- **Mistake Clinic.** 12 common mistakes: the wrong work, why it is tempting, and the fix.

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